MODULE-I

GAS POWER CYCLES

1.1 Theoretical Analysis

The accurate analysis of the various processes taking place in an internal combustion engine is a very complex problem. If these processes were to be analyzed experimentally, the analysis would be very realistic no doubt. It would also be quite accurate if the tests are carried out correctly and systematically, but it would be time consuming. If a detailed analysis has to be carried out involving changes in operating parameters, the cost of such an analysis would be quite high, even prohibitive. An obvious solution would be to look for a quicker and less expensive way of studying the engine performance characteristics. A theoretical analysis is the obvious answer.

A theoretical analysis, as the name suggests, involves analyzing the engine performance without actually building and physically testing an engine. It involves *simulating* an engine operation with the help of thermodynamics so as to formulate mathematical expressions which can then be solved in order to obtain the relevant information. The method of solution will depend upon the complexity of the formulation of the mathematical expressions which in turn will depend upon the assumptions that have been introduced in order to analyze the processes in the engine. The more the assumptions, the simpler will be the mathematical expressions and the easier the calculations, but the lesser will be the accuracy of the final results.

The simplest theoretical analysis involves the use of the air standard cycle, which has the largest number of simplifying assumptions.

1.2 A Thermodynamic Cycle

In some practical applications, notably steam power and refrigeration, a thermodynamic cycle can be identified.

A thermodynamic cycle occurs when the working fluid of a system experiences a number of processes that eventually return the fluid to its initial state.

In steam power plants, water is pumped (for which work W_P is required) into a boiler and evaporated into steam while heat Q_A is supplied at a high temperature. The steam flows through a turbine doing work W_T and then passes into a condenser where it is condensed into water with consequent rejection of heat Q_R to the atmosphere. Since the water is returned to its initial state, the net change in energy is zero, assuming no loss of water through leakage or evaporation.

An energy equation pertaining only to the system can be derived. Considering a system with one entering and one leaving flow stream for the time period t_1 to t_2

$$\Delta Q - \Delta W + \Delta E_{f_{in}} - \Delta E_{f_{out}} = \Delta E_{system} \qquad (1)$$

ΔQ is the heat transfer across the boundary, +ve for heat *added to* the system and
–ve for heat *taken from* the system.

ΔW is the work transfer across the boundary, +ve for work *done by* the system and
-ve for work *added to* the system

$\Delta E_{f_{in}}$ is the energy of all forms *carried* by the fluid across the boundary *into* the system

$\Delta E_{f_{out}}$ is the energy of all forms *carried* by the fluid across the boundary *out* of system

ΔE_{system} is the energy of all forms *stored* within the system, +ve for energy *increase*
-ve for energy *decrease*

In the case of the steam power system described above

$$Q_A + Q_R = \sum Q = \sum W = W_T + W_P \qquad (2)$$

All thermodynamic cycles have a heat rejection process as an invariable characteristic and the net work done is always less than the heat supplied, although, as shown in Eq. 2, it is equal to the sum of heat added and the heat rejected (Q_R is a negative number).

The thermal efficiency of a cycle, η_{th}, is defined as the fraction of heat supplied to a thermodynamic cycle that is converted to work, that is

$$\eta_{th} = \frac{\sum W}{Q_A}$$

$$= \frac{Q_A + Q_R}{Q_A} \qquad (3)$$

This efficiency is sometimes confused with the enthalpy efficiency, η_e, or the fuel conversion efficiency, η_f

$$\eta_e = \frac{\sum W}{m_f Q_c} \qquad (4)$$

This definition applies to combustion engines which have as a source of energy the chemical energy residing in a fuel used in the engine.

Any device that operated in a thermodynamic cycle, absorbs thermal energy from a source, rejects a part of it to a sink and presents the difference between the energy absorbed and energy rejected *as work to the surroundings* is called a heat engine.

A heat engine is, thus, a device that produces work. In order to achieve this purpose, the heat engine uses a certain working medium which undergoes the following processes:

1. A compression process where the working medium absorbs energy as work.
2. A heat addition process where the working medium absorbs energy as heat from a source.
3 An expansion process where the working medium transfers energy as work to the surroundings.
4. A heat rejection process where the working medium rejects energy as heat to a sink.

If the working medium does not undergo any change of phase during its passage through the cycle, the heat engine is said to operate in a non-phase change cycle. A phase change cycle is one in which the working medium undergoes changes of phase. The air standard cycles, using air as the working medium are examples of non-phase change cycles while the steam and vapor compression refrigeration cycles are examples of phase change cycles.

1.3 Air Standard Cycles

The air standard cycle is a cycle followed by a heat engine which uses air as the working medium. Since the air standard analysis is the simplest and most idealistic, such cycles are also called *ideal cycles* and the engine running on such cycles are called *ideal engines.*

In order that the analysis is made as simple as possible, certain assumptions have to be made. These assumptions result in an analysis that is far from correct for most actual combustion engine processes, but the analysis is of considerable value for indicating the upper limit of performance. The analysis is also a simple means for indicating the relative effects of principal variables of the cycle and the relative size of the apparatus.

Assumptions

1. The working medium is a perfect gas with constant specific heats and molecular weight corresponding to values at room temperature.
2. No chemical reactions occur during the cycle. The heat addition and heat rejection processes are merely heat transfer processes.
3. The processes are reversible.
4. Losses by heat transfer from the apparatus to the atmosphere are assumed to be zero in this analysis.
5. The working medium at the end of the process (cycle) is unchanged and is at the same condition as at the beginning of the process (cycle).

In The selecting an idealized process one is always faced with the fact that the simpler the assumptions, the easier the analysis, but the farther the result from reality. The air cycle has the advantage of being based on a few simple assumptions and of lending itself to rapid and easy mathematical handling without recourse to thermodynamic charts or tables or complicated calculations. On the other hand, there is always the danger of losing sight of its limitations and of trying to employ it beyond its real usefulness.

Equivalent Air Cycle

A particular air cycle is usually taken to represent an approximation of some real set of processes which the user has in mind. Generally speaking, the air cycle representing a given real cycle is called an *equivalent air cycle*. The equivalent cycle has, in general, the following characteristics in common with the real cycle which it approximates:

1. A similar sequence of processes.
2. Same ratio of maximum to minimum volume for reciprocating engines or maximum to minimum pressure for gas turbine engines.
3. The same pressure and temperature at a given reference point.
4. An appropriate value of heat addition per unit mass of air.

1.4 The Carnot Cycle

This cycle was proposed by Sadi Carnot in 1824 and has the highest possible efficiency for any cycle. Figures 1 and 2 show the P-V and T-s diagrams of the cycle.

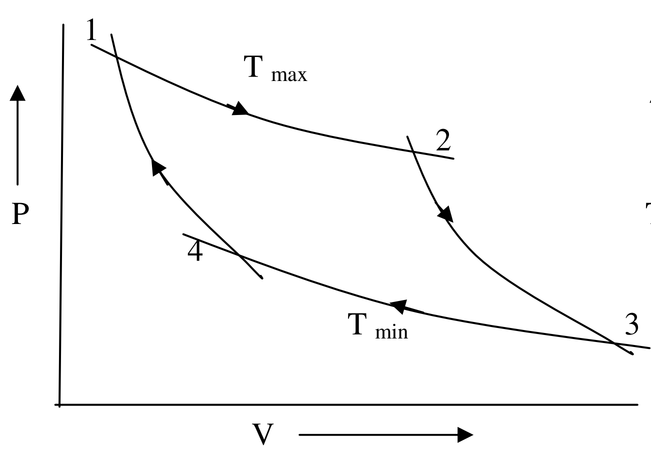


Fig.1: P-V Diagram of Carnot Cycle.

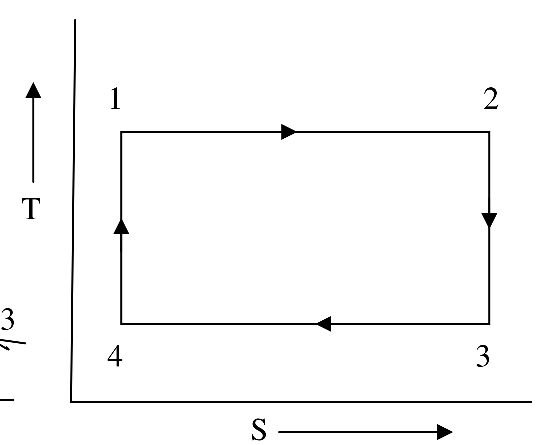


Fig.2: T-S Diagram of Carnot Cycle.

Assuming that the charge is introduced into the engine at point 1, it undergoes isentropic compression from 4 to 1. The temperature of the charge rises from T_{min} to T_{max}. At point 2, heat is added isothermally. This causes the air to expand, forcing the piston forward, thus doing work on the piston. At point 3, the source of heat is removed at constant temperature. At point 4, a cold body is applied to the end of the cylinder and the piston reverses, thus compressing the air isothermally; heat is rejected to the cold body. At point 1, the cold body is removed and the charge is compressed isentropically till it reaches a temperature T_{max} once again. Thus, the heat addition and rejection processes are isothermal while the compression and expansion processes are isentropic.

From thermodynamics, per unit mass of charge

$$\text{Heat supplied from point 1 to 2} = p_2 v_2 \ln \frac{v_2}{v_1} \qquad (5)$$

$$\text{Heat rejected from point 3 to 4} = p_3 v_3 \ln \frac{v_4}{v_3} \qquad (6)$$

$$\text{Now } p_2 v_2 = RT_{max} \qquad (7)$$

$$\text{And } p_4 v_4 = RT_{min} \qquad (8)$$

Since Work done, per unit mass of charge, W = heat supplied – heat rejected

$$W = RT_{\max} \ln \frac{v_3}{v_2} - RT_{\min} \ln \frac{v_1}{v_4}$$

$$= R \ln(r)(T_{\max} - T_{\min}) \qquad (9)$$

We have assumed that the compression and expansion ratios are equal, that is

$$\frac{v_3}{v_2} = \frac{v_1}{v_4} \qquad (10)$$

$$\text{Heat supplied } Q_s = R\, T_{max} \ln(r) \qquad (11)$$

Hence, the thermal efficiency of the cycle is given by

$$\eta_{th} = \frac{R \ln(r)(T_{\max} - T_{\min})}{R \ln(r) T_{\max}}$$

$$= \frac{T_{\max} - T_{\min}}{T_{\max}} \qquad (12)$$

From Eq. 12 it is seen that the thermal efficiency of the Carnot cycle is only a function of the maximum and minimum temperatures of the cycle. The efficiency will increase if the minimum temperature (or the temperature at which the heat is rejected) is as low as possible.

According to this equation, the efficiency will be equal to 1 if the minimum temperature is zero, which happens to be the absolute zero temperature in the thermodynamic scale.

This equation also indicates that for optimum (Carnot) efficiency, the cycle (and hence the heat engine) must operate between the limits of the highest and lowest possible temperatures. In other words, the engine should take in all the heat at as high a temperature as possible and should reject the heat at as low a temperature as possible. For the first condition to be achieved, combustion (as applicable for a real engine using fuel to provide heat) should begin at the highest possible temperature, for then the irreversibility of the chemical reaction would be reduced. Moreover, in the cycle, the expansion should proceed to the lowest possible temperature in order to obtain the maximum amount of work. These conditions are the aims of all designers of modern heat engines. The conditions of heat rejection are governed, in practice, by the temperature of the atmosphere.

It is impossible to construct an engine which will work on the Carnot cycle. In such an engine, it would be necessary for the piston to move very slowly during the first part of the forward stroke so that it can follow an isothermal process. During the remainder of the forward stroke, the piston would need to move very quickly as it has to follow an isentropic process. This variation in the speed of the piston cannot be achieved in practice. Also, a very long piston stroke would produce only a small amount of work most of which would be absorbed by the friction of the moving parts of the engine.

Since the efficiency of the cycle, as given by Eq. 11, is dependent only on the maximum and minimum temperatures, it does not depend on the working medium. It is thus independent of the properties of the working medium.

1.5 The Otto Cycle

The Otto cycle, which was first proposed by a Frenchman, Beau de Rochas in 1862, was first used on an engine built by a German, Nicholas A. Otto, in 1876. The cycle is also called a *constant volume* or *explosion* cycle. This is the equivalent air cycle for reciprocating piston engines using spark ignition. Figures 5 and 6 show the P-V and T-s diagrams respectively.

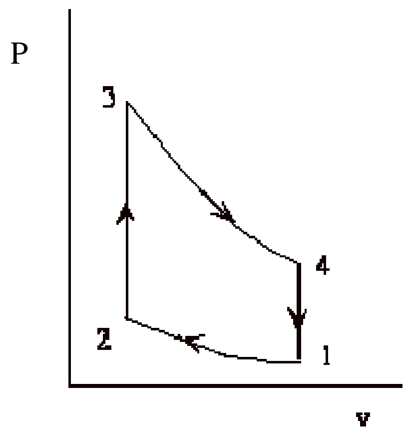


Fig.3: P-V Diagram of Otto Cycle.

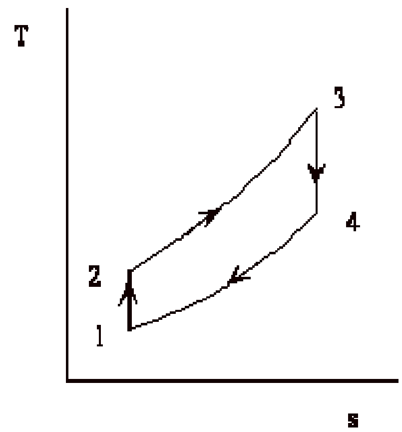


Fig.4: T-S Diagram of Otto Cycle.

At the start of the cycle, the cylinder contains a mass M of air at the pressure and volume indicated at point 1. The piston is at its lowest position. It moves upward and the gas is compressed isentropically to point 2. At this point, heat is added at constant volume which raises the pressure to point 3. The high pressure charge now expands isentropically, pushing the piston down on its expansion stroke to point 4 where the charge rejects heat at constant volume to the initial state, point 1.

The isothermal heat addition and rejection of the Carnot cycle are replaced by the constant volume processes which are, theoretically more plausible, although in practice, even these processes are not practicable.

The heat supplied, Q_s, per unit mass of charge, is given by

$$c_v(T_3 - T_2) \qquad (13)$$

the heat rejected, Q_r per unit mass of charge is given by

$$c_v(T_4 - T_1) \qquad (14)$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1}{T_2}\left\{\frac{\left(\frac{T_4}{T_1} - 1\right)}{\left(\frac{T_3}{T_2} - 1\right)}\right\} \qquad (15)$$

Now $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \frac{T_4}{T_3}$

And since $\frac{T_1}{T_2} = \frac{T_4}{T_3}$ *we have* $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

Hence, substituting in Eq. 15, we get, assuming that r is the compression ratio V_1/V_2

$$\eta_{th} = 1 - \frac{T_1}{T_2}$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$= 1 - \frac{1}{r^{\gamma-1}} \qquad (16)$$

In a true thermodynamic cycle, the term *expansion ratio* and *compression ratio* are synonymous. However, in a real engine, these two ratios need not be equal because of the valve timing and therefore the term *expansion ratio* is preferred sometimes.

Equation 16 shows that the thermal efficiency of the theoretical Otto cycle increases with increase in compression ratio and specific heat ratio but is independent of the heat added (independent of load) and initial conditions of pressure, volume and temperature.

Figure 5 shows a plot of thermal efficiency versus compression ratio for an Otto cycle. It is seen that the increase in efficiency is significant at lower compression ratios. This is also seen in Table 1 given below.

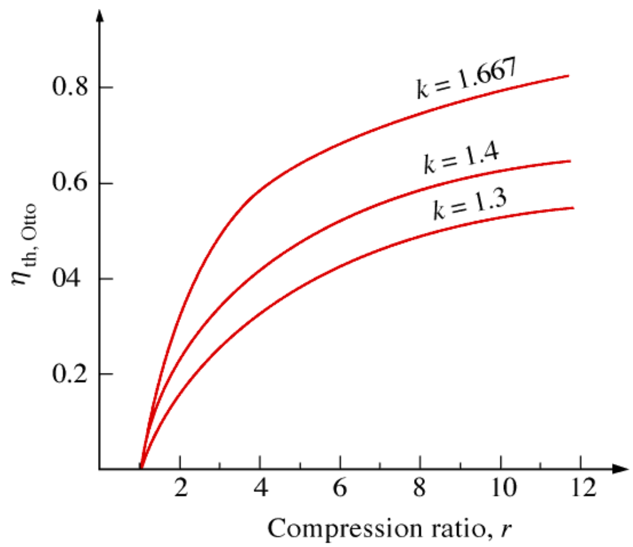


Fig.5: variation of efficiency with compression ratio

Table1: compression ratio and corresponding thermal efficiency for Otto cycle

R	η
1	0
2	0.242
3	0.356
4	0.426
5	0.475
6	0.512
7	0.541
8	0.565
9	0.585
10	0.602
16	0.67
20	0.698
50	0.791

From the table it is seen that if:

CR is increased from 2 to 4, efficiency increase is 76%

CR is increased from 4 to 8, efficiency increase is only 32.6%

CR is increased from 8 to 16, efficiency increase is only 18.6%

Mean effective pressure:

It is seen that the air standard efficiency of the Otto cycle depends only on the compression ratio. However, the pressures and temperatures at the various points in the cycle and the net work done, all depend upon the initial pressure and temperature and the heat input from point 2 to point 3, besides the compression ratio.

A quantity of special interest in reciprocating engine analysis is the mean effective pressure. Mathematically, it is the net work done on the piston, W, divided by the piston *displacement* volume, $V_1 - V_2$. This quantity has the units of pressure. Physically, it is that constant pressure which, if exerted on the piston for the whole outward stroke, would yield work equal to the work of the cycle. It is given by

$$mep = \frac{W}{V_1 - V_2}$$

$$= \frac{\eta Q_{2-3}}{V_1 - V_2} \qquad (17)$$

where Q_{2-3} is the heat added from points 2 to 3.

Work done per kg of air

$$W = \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1} = mepV_s = P_m(V_1 - V_2)$$

$$mep = \frac{1}{(V_1 - V_2)}\left[\frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1}\right] \qquad (17A)$$

The pressure ratio P_3/P_2 is known as explosion ratio r_p

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\nu} = r^{\nu} \Rightarrow P_2 = P_1 r^{\nu},$$

$$P_3 = P_2 r_p = P_1 r^{\nu} r_p,$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\nu} = P_1 r^{\nu} r_p \left(\frac{V_2}{V_1}\right)^{\nu} = P_1 r_p$$

$$\frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = r$$
$$\therefore V_s = V_c (r-1)$$

Substituting the above values in Eq 17A

$$mep = P_1 \frac{r(r_p - 1)(r^{\gamma-1} - 1)}{(r-1)(\gamma-1)}$$

Now

$$V_1 - V_2 = V_1 \left(1 - \frac{V_2}{V_1}\right)$$

$$= V_1 \left(1 - \frac{1}{r}\right) \qquad (18)$$

Here r is the compression ratio, V_1/V_2

From the equation of state:

$$V_1 = M \frac{R_0}{m} \frac{T_1}{p_1} \qquad (19)$$

R_0 is the universal gas constant

Substituting for V_1 from Eq. 3 in Eq. 2 and then substituting for $V_1 - V_2$ in Eq. 1 we get

$$mep = \eta \frac{Q_{2-3} \frac{p_1 m}{M R_0 T_1}}{1 - \frac{1}{r}} \qquad (20)$$

The quantity Q_{2-3}/M is the heat added between points 2 and 3 *per unit mass* of air (M is the mass of air and m is the molecular weight of air); and is denoted by Q', thus

$$mep = \eta \frac{Q' \frac{p_1 m}{R_0 T_1}}{1 - \frac{1}{r}} \qquad (21)$$

We can non-dimensionalize the mep by dividing it by p_1 so that we can obtain the following equation

$$\frac{mep}{p_1} = \eta \left[\frac{1}{1 - \frac{1}{r}} \right] \left[\frac{Q' m}{R_0 T_1} \right] \qquad (22)$$

Since $\frac{R_0}{m} = c_v(\gamma - 1)$, we can substitute it in Eq. 25 to get

$$\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_1} \frac{1}{\left[1 - \frac{1}{r}\right][\gamma - 1]} \qquad (23)$$

The dimensionless quantity mep/p_1 is a function of the heat added, initial temperature, compression ratio and the properties of air, namely, c_v and γ. We see that the mean effective pressure is directly proportional to the heat added and inversely proportional to the initial (or ambient) temperature.

We can substitute the value of η from Eq. 20 in Eq. 26 and obtain the value of mep/p_1 for the Otto cycle in terms of the compression ratio and heat added.

In terms of the pressure ratio, p_3/p_2 denoted by r_p we could obtain the value of mep/p_1 as follows:

$$\frac{mep}{p_1} = \frac{r(r_p - 1)(r^{\gamma - 1} - 1)}{(r - 1)(\gamma - 1)} \qquad (24)$$

We can obtain a value of r_p in terms of Q' as follows:

$$r_p = \frac{Q'}{c_v T_1 r^{\gamma - 1}} + 1 \qquad (25)$$

Choice of Q'

We have said that

$$Q' = \frac{Q_{2-3}}{M} \qquad (26)$$

M is the mass of charge (air) per cycle, kg.

Now, in an actual engine

$$Q_{2-3} = M_f Q_c$$

$$= FM_a Q_c \ in\ kJ/cycle \qquad (27)$$

M_f is the mass of fuel supplied per cycle, kg

Q_c is the heating value of the fuel, kJ/kg

M_a is the mass of air taken in per cycle

F is the fuel air ratio = M_f/M_a

Substituting for Eq. (B) in Eq. (A) we get

$$Q' = \frac{FM_a Q_c}{M} \qquad (28)$$

$$Now \quad \frac{M_a}{M} \approx \frac{V_1 - V_2}{V_1}$$

$$And \quad \frac{V_1 - V_2}{V_1} = 1 - \frac{1}{r} \qquad (29)$$

So, substituting for M_a/M from Eq. (33) in Eq. (32) we get

$$Q' = FQ_c\left(1 - \frac{1}{r}\right) \qquad (30)$$

For isooctane, FQ_c at stoichiometric conditions is equal to 2975 kJ/kg, thus

$$Q' = 2975(r - 1)/r \qquad (31)$$

At an ambient temperature, T_1 of 300K and c_v for air is assumed to be 0.718 kJ/kgK, we get a value of $Q'/c_vT_1 = 13.8(r - 1)/r$.

Under fuel rich conditions, $\varphi = 1.2$, $Q'/ c_vT_1 = 16.6(r - 1)/r$. (32)

Under fuel lean conditions, $\varphi = 0.8$, $Q'/ c_vT_1 = 11.1(r - 1)/r$ (33)

1.6 The Diesel Cycle

This cycle, proposed by a German engineer, Dr. Rudolph Diesel to describe the processes of his engine, is also called the constant pressure cycle. This is believed to be the equivalent air cycle for the reciprocating slow speed compression ignition engine. The P-V and T-s diagrams are shown in Figs 6and 7 respectively.

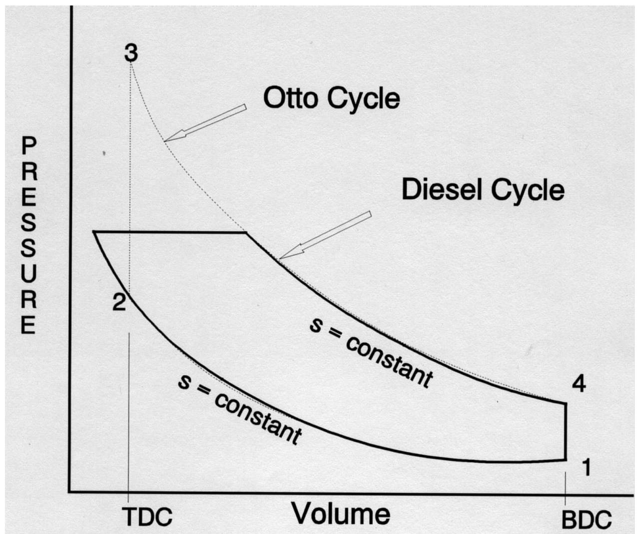


Fig.6: P-V Diagram of Diesel Cycle.

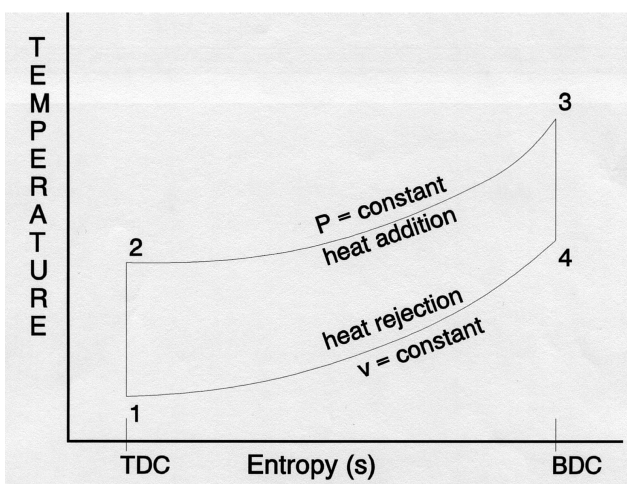


Fig.7: T-S Diagram of Diesel Cycle.

The cycle has processes which are the same as that of the Otto cycle except that the heat is added at constant pressure.

The heat supplied, Q_s is given by

$$c_p(T_3 - T_2) \tag{34}$$

whereas the heat rejected, Q_r is given by

$$c_v(T_4 - T_1) \tag{35}$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)}\right\} \tag{36}$$

From the T-s diagram, Fig. 7, the difference in enthalpy between points 2 and 3 is the same as that between 4 and 1, thus

$$\Delta s_{2-3} = \Delta s_{4-1}$$

$$\therefore\ c_v \ln\left(\frac{T_4}{T_1}\right) = c_p \ln\left(\frac{T_3}{T_2}\right)$$

$$\therefore\ \ln\left(\frac{T_4}{T_1}\right) = \gamma \ln\left(\frac{T_3}{T_2}\right)$$

$$\therefore\ \frac{T_4}{T_1} = \left(\frac{T_3}{T_2}\right)^{\gamma} \text{ and } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \frac{1}{r^{\gamma - 1}}$$

Substituting in eq. 36, we get

$$\eta_{th} = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\left(\frac{T_3}{T_2}\right)^{\gamma} - 1}{\frac{T_3}{T_2} - 1}\right] \qquad (37)$$

$$\text{Now } \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c = cut-off \;\; ratio$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right] \qquad (38)$$

When Eq. 38 is compared with Eq. 20, it is seen that the expressions are similar except for the term in the parentheses for the Diesel cycle. It can be shown that this term is always greater than unity.

Now $r_c = \frac{V_3}{V_2} = \frac{V_3}{V_4} \bigg/ \frac{V_2}{V_1} = \frac{r}{r_e}$ where r is the compression ratio and r_e is the expansion ratio

Thus, the thermal efficiency of the Diesel cycle can be written as

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\left(\frac{r}{r_e}\right)^{\gamma} - 1}{\gamma\left(\frac{r}{r_e} - 1\right)}\right] \qquad (39)$$

Let $r_e = r - \Delta$ since r is greater than r_e. Here, Δ is a small quantity. We therefore have

$$\frac{r}{r_e} = \frac{r}{r - \Delta} = \frac{r}{r\left(1 - \frac{\Delta}{r}\right)} = \left(1 - \frac{\Delta}{r}\right)^{-1}$$

We can expand the last term binomially so that

$$\left(1 - \frac{\Delta}{r}\right)^{-1} = 1 + \frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \cdots$$

$$\text{Also}\ \left(\frac{r}{r_e}\right)^{\gamma} = \frac{r^{\gamma}}{(r-\Delta)^{\gamma}} = \frac{r^{\gamma}}{r^{\gamma}\left(1-\dfrac{\Delta}{r}\right)^{\gamma}} = \left(1-\frac{\Delta}{r}\right)^{-\gamma}$$

We can expand the last term binomially so that

$$\left(1-\frac{\Delta}{r}\right)^{-\gamma} = 1 + \gamma\frac{\Delta}{r} + \frac{\gamma(\gamma+1)}{2!}\frac{\Delta^2}{r^2} + \frac{\gamma(\gamma+1)(\gamma+2)}{3!}\frac{\Delta^3}{r^3} + \cdots$$

Substituting in Eq. 39, we get

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\dfrac{\Delta}{r} + \dfrac{(\gamma+1)}{2!}\dfrac{\Delta^2}{r^2} + \dfrac{(\gamma+1)(\gamma+2)}{3!}\dfrac{\Delta^3}{r^3} + \cdots}{\dfrac{\Delta}{r} + \dfrac{\Delta^2}{r^2} + \dfrac{\Delta^3}{r^3} + \cdots}\right] \qquad (40)$$

Since the coefficients of $\frac{\Delta}{r}, \frac{\Delta^2}{r^r}, \frac{\Delta^3}{r^3}$, etc are greater than unity, the quantity in the brackets in Eq. 40 will be greater than unity. Hence, for the Diesel cycle, we subtract $\frac{1}{r^{\gamma-1}}$ times a quantity greater than unity from one, hence for the same r, the Otto cycle efficiency is greater than that for a Diesel cycle.

If $\frac{\Delta}{r}$ is small, the square, cube, etc of this quantity becomes progressively smaller, so the thermal efficiency of the Diesel cycle will tend towards that of the Otto cycle.

From the foregoing we can see the importance of cutting off the fuel supply early in the forward stroke, a condition which, because of the short time available and the high pressures involved, introduces practical difficulties with high speed engines and necessitates very rigid fuel injection gear.

In practice, the diesel engine shows a better efficiency than the Otto cycle engine because the compression of air alone in the former allows a greater compression ratio to be employed. With a mixture of fuel and air, as in practical Otto cycle engines, the maximum temperature developed by compression must not exceed the self ignition temperature of the mixture; hence a definite limit is imposed on the maximum value of the compression ratio.

Thus Otto cycle engines have compression ratios in the range of 7 to 12 while diesel cycle engines have compression ratios in the range of 16 to 22.

$$mep = \frac{1}{V_s}\left[P_2\left(V_3 - V_2\right) + \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1}\right] \qquad (42)$$

The pressure ratio P_3/P_2 is known as explosion ratio r_p

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\nu} = r^{\nu} \Rightarrow P_2 = P_1 r^{\nu},$$

$$P_3 = P_2 = P_1 r^{\nu}$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\nu} = P_1 r^{\nu}\left(\frac{V_2}{V_1}\right)^{\nu} = P_1 r_c^{\ \nu}$$

$$V_4 = V_1, V_2 = V_c,$$

$$\frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = r$$

$$\therefore V_s = V_c\left(r - 1\right)$$

Substituting the above values in Eq 42 to get Eq (42A)

In terms of the cut-off ratio, we can obtain another expression for mep/p_1 as follows

$$mep = P_1 \frac{\gamma r^{\gamma}\left(r_c - 1\right) - r\left(r_c^{\gamma} - 1\right)}{\left(r - 1\right)\left(\gamma - 1\right)} \qquad (42A)$$

We can obtain a value of r_c for a Diesel cycle in terms of Q' as follows:

$$r_c = \frac{Q'}{c_p T_1 r^{\gamma - 1}} + 1 \qquad (41)$$

We can substitute the value of η from Eq. 38 in Eq. 26, reproduced below and obtain the value of mep/p_1 for the Diesel cycle.

$$\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_1} \frac{1}{\left[1 - \frac{1}{r}\right][\gamma - 1]}$$

For the Diesel cycle, the expression for mep/p_3 is as follows:

$$\frac{mep}{p_3} = \frac{mep}{p_1}\left(\frac{1}{r^\gamma}\right) \qquad (43)$$

Modern high speed diesel engines do not follow the Diesel cycle. The process of heat addition is partly at constant volume and partly at constant pressure. This brings us to the dual cycle.

1.7 The Dual Cycle

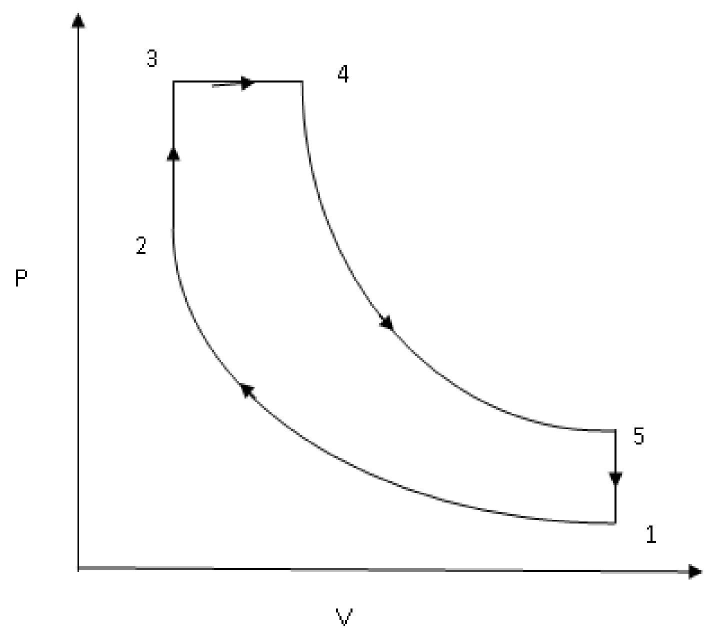


Fig.8: P-V Diagram of Dual Cycle.

Process 1-2: Reversible adiabatic compression.

Process 2-3: Constant volume heat addition.

Process 3-4: Constant pressure heat addition.

Process 4-5: Reversible adiabatic expansion.

Process 5-1: Constant volume heat reject

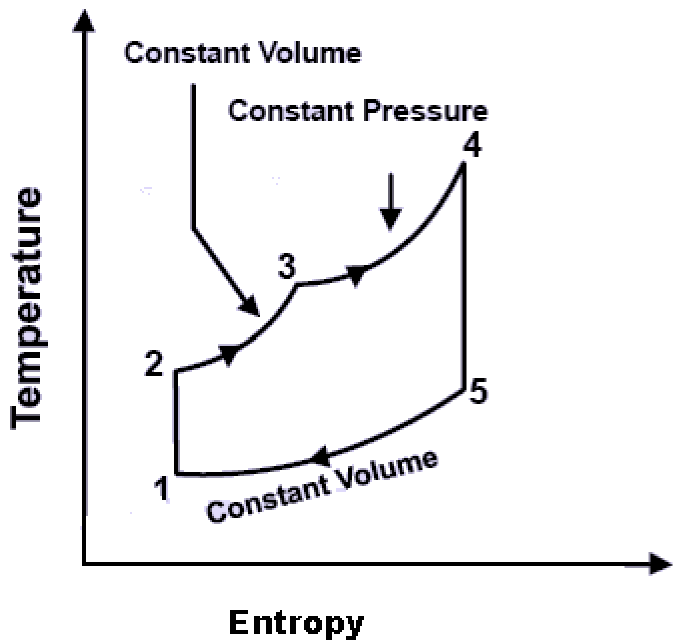


Fig.9: T-S Diagram of Carnot Cycle.

An important characteristic of real cycles is the ratio of the mean effective pressure to the maximum pressure, since the mean effective pressure represents the useful (average) pressure acting on the piston while the maximum pressure represents the pressure which chiefly affects the strength required of the engine structure. In the constant-volume cycle, shown in Fig. 8, it is seen that the quantity mep/p_3 falls off rapidly as the compression ratio increases, which means that for a given mean effective pressure the maximum pressure rises rapidly as the compression ratio increases. For example, for a mean effective pressure of 7 bar and Q'/c_vT_1 of 12, the maximum pressure at a compression ratio of 5 is 28 bar whereas at a compression ratio of 10, it rises to about 52 bar. Real cycles follow the same trend and it becomes a practical necessity to limit the maximum pressure when high compression ratios are used, as in diesel engines. This also indicates that diesel engines will have to be stronger (and hence heavier) because it has to withstand higher peak pressures.

Constant pressure heat addition achieves rather low peak pressures unless the compression ratio is quite high. In a real diesel engine, in order that combustion takes place at constant pressure, fuel has to be injected very late in the compression stroke (practically at the top dead center). But in order to increase the efficiency of the cycle, the fuel supply must be cut off early in the expansion stroke, both to give sufficient time for the fuel to burn and thereby increase combustion efficiency and reduce after burning but also reduce emissions. Such situations can be achieved if the engine was a slow speed type so that the piston would move sufficiently slowly for combustion to take place despite the late injection of the fuel. For modern high speed compression ignition engines it is not possible to achieve constant pressure combustion. Fuel is injected somewhat earlier in the compression stroke and has to go through the various stages of combustion. Thus it is seen that combustion is nearly at constant volume (like in a spark ignition engine). But the peak pressure is limited because of strength considerations so the rest of the heat addition is believed to take place at constant pressure in a cycle. This has led to the formulation of the dual combustion cycle. In this cycle, for high compression ratios, the peak pressure is not allowed to increase beyond a certain limit and to account for the total addition, the rest of the heat is assumed to be added at constant pressure. Hence the name *limited pressure cycle.*

The cycle is the equivalent air cycle for reciprocating high speed compression ignition engines. The P-V and T-s diagrams are shown in Figs.8 and 9. In the cycle, compression and expansion processes are isentropic; heat addition is partly at constant volume and partly at constant pressure while heat rejection is at constant volume as in the case of the Otto and Diesel cycles.

The heat supplied, Q_s per unit mass of charge is given by

$$c_v(T_3 - T_2) + c_p(T_{3'} - T_2) \qquad (44)$$

whereas the heat rejected, Q_r per unit mass of charge is given by

$$c_v(T_4 - T_1)$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v\left(T_4 - T_1\right)}{c_v\left(T_3 - T_2\right) + c_p\left(T_{3'} - T_2\right)} \qquad (44A)$$

$$= 1 - \left\{ \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right) + \gamma T_3\left(\frac{T_{3'}}{T_3} - 1\right)} \right\} \qquad (44B)$$

$$= 1 - \frac{\frac{T_4}{T_1} - 1}{\frac{T_2}{T_1}\left(\frac{T_3}{T_2} - 1\right) + \frac{\gamma T_3}{T_2}\frac{T_2}{T_1}\left(\frac{T_{3'}}{T_3} - 1\right)} \qquad (44C)$$

From thermodynamics

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = r_p \qquad (45)$$

the explosion or pressure ratio

and

$$\frac{T_{3'}}{T_3} = \frac{V_{3'}}{V_3} = r_c \qquad (46)$$

the cut-off ratio.

$$\text{Now, } \frac{T_4}{T_1} = \frac{p_4}{p_1} = \frac{p_4}{p_{3'}}\frac{p_{3'}}{p_3}\frac{p_3}{p_2}\frac{p_2}{p_1}$$

$$\text{Also } \frac{p_4}{p_{3'}} = \left(\frac{V_{3'}}{V_4}\right)^{\gamma} = \left(\frac{V_{3'}}{V_3}\frac{V_3}{V_4}\right)^{\gamma} = \left(r_c \frac{1}{r}\right)^{\gamma}$$

$$\text{And } \frac{p_2}{p_1} = r^{\gamma}$$

$$\text{Thus } \frac{T_4}{T_1} = r_p r_c^{\gamma}$$

$$\text{Also } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = r^{\gamma - 1}$$

Therefore, the thermal efficiency of the dual cycle is

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_p r_c^{\gamma} - 1}{(r_p - 1) + \gamma r_p (r_c - 1)}\right] \qquad (46)$$

We can substitute the value of η from Eq. 46 in Eq. 26 and obtain the value of mep/p_1 for the dual cycle.

In terms of the cut-off ratio and pressure ratio, we can obtain another expression for mep/p_1 as follows:

$$\frac{mep}{p_1} = \frac{\gamma r_p r^{\gamma}(r_c - 1) + r^{\gamma}(r_p - 1) - r(r_p r_c^{\gamma} - 1)}{(r-1)(\gamma - 1)} \qquad (47)$$

For the dual cycle, the expression for mep/p_3 is as follows:

$$\frac{mep}{p_3} = \frac{mep}{p_1}\left(\frac{p_1}{p_3}\right) \qquad (48)$$

Since the dual cycle is also called the limited pressure cycle, the peak pressure, **p_3, is usually specified. Since the initial pressure, p_1, is known, the ratio p_3/p_1 is known.** We can correlate r_p with this ratio as follows:

$$r_p = \frac{p_3}{p_1}\left(\frac{1}{r^{\gamma}}\right) \qquad (49)$$

We can obtain an expression for r_c in terms of Q' and r_p and other known quantities as follows:

$$r_c = \frac{1}{\gamma}\left(\left[\left\{\frac{Q'}{c_v T_1 r^{\gamma-1}}\right\}\frac{1}{r_p}\right] + (\gamma - 1)\right) \qquad (50)$$

We can also obtain an expression for r_p in terms of Q' and r_c and other known quantities as follows:

$$r_p = \frac{\left[\dfrac{Q'}{c_v T_1 r^{\gamma - 1}} + 1 \right]}{1 + \gamma r_c - \gamma} \qquad (51)$$

1.8 Stirling cycle

When a confined body of gas (air, helium, whatever) is heated, its pressure rises. This increased pressure can push on a piston and do work. The body of gas is then cooled, pressure drops, and the piston can return. The same cycle repeats over and over, using the same body of gas. That is all there is to it. No ignition, no carburetion, no valve train, no explosions. Many people have a hard time understanding the Stirling because it is so much simpler than conventional internal combustion engines.

The Stirling cycle is described using the pressure-volume (P-v) and temperature-entropy (T-s) diagrams shown in Figure 1. The P-v and T-s diagrams show the state of a "working fluid" at any point during the idealized cycle. The working fluid is normally a gas...in the Stirling engines being produced to us, the working fluid is air.

In the idealized Stirling cycle heat (i.e., energy) is transferred to the working fluid during the segment 2-3-4. Conversely, heat (energy) is extracted from the working fluid during the segment 4-1-2. During segment 2-3 heat is transferred to the fluid internally via regeneration of the energy transferred from the fluid during segment 4-1. The means that (ideally) heat is added from an external source only during segment 3-4, and that heat is rejected to the surrounding environment only during segment 1-2. Note that this is the idealized cycle.

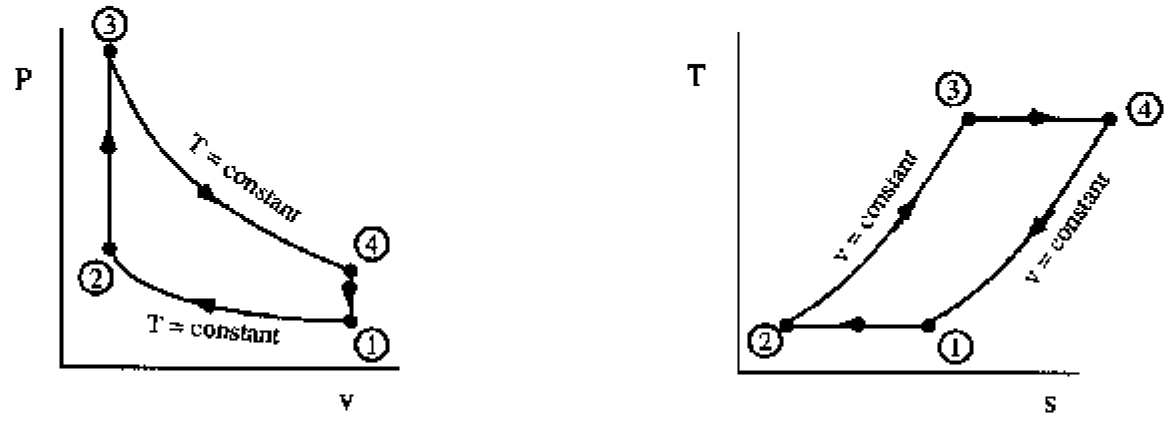
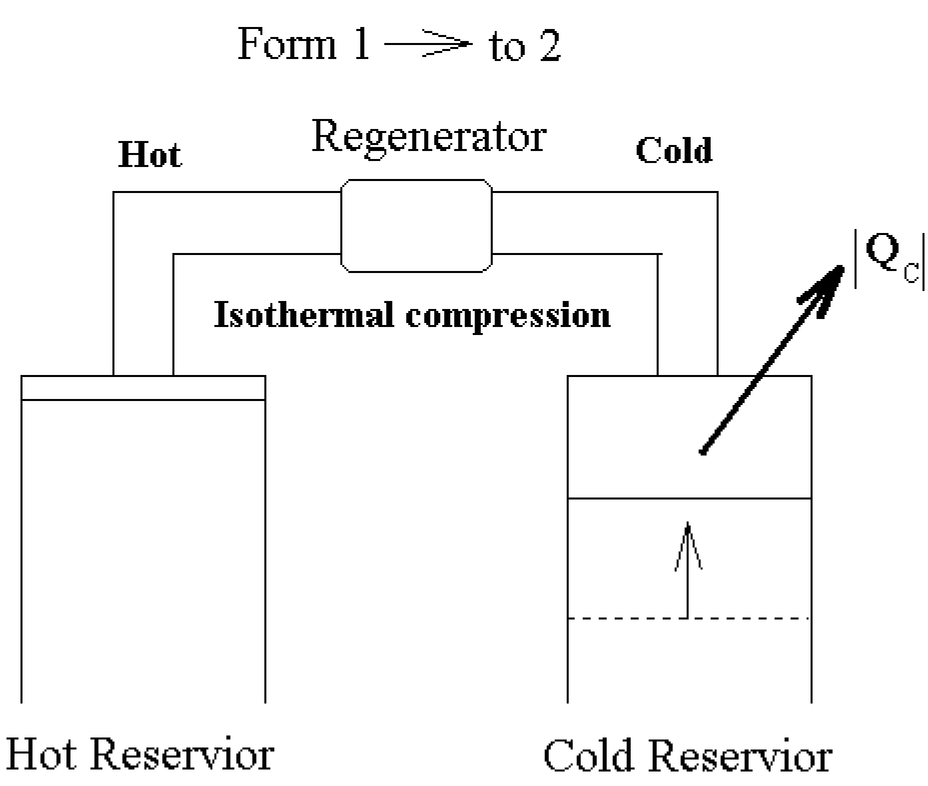


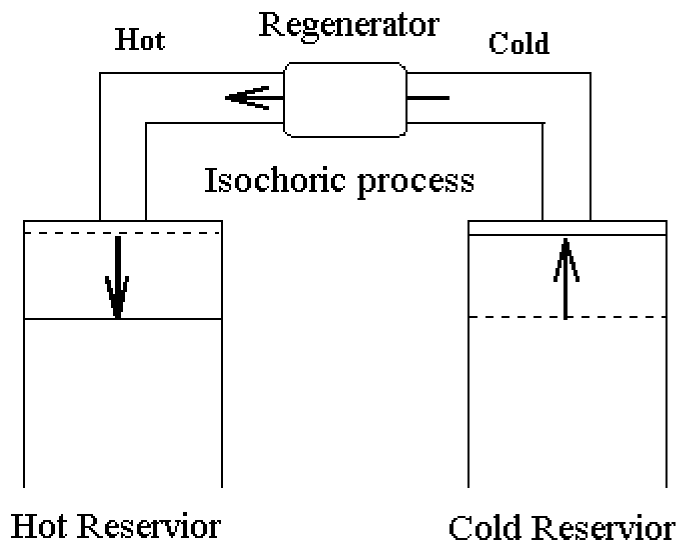
Figure 1: P-v and T-s diagrams used to define the idealized Stirling cycle



1→2 It is an isothermal process, the piston in contact with cold reservoir is compressed isothermally, hence heat |Qc| has been rejected, and (isothermal compression → dU = 0, W is positive and Qc is negative) the heat rejected is

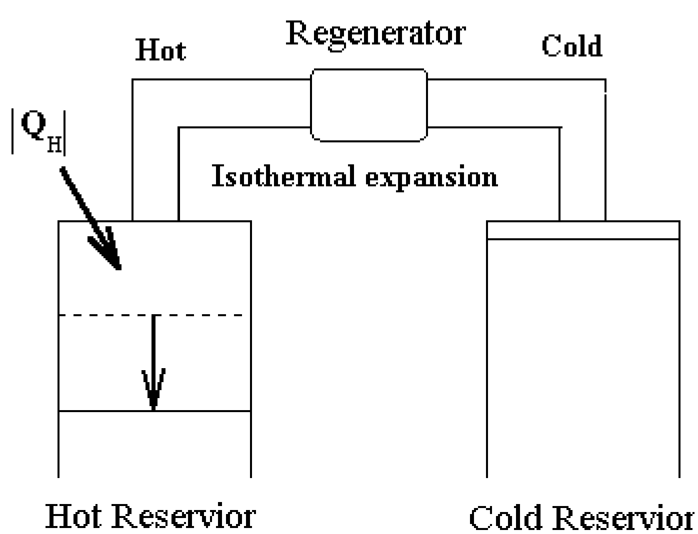
$$Q_c = P_1 V_1 \ln \frac{V_1}{V_2} = RT_c \ln(r_c) \qquad 52$$

Form 2 ⟶ to 3



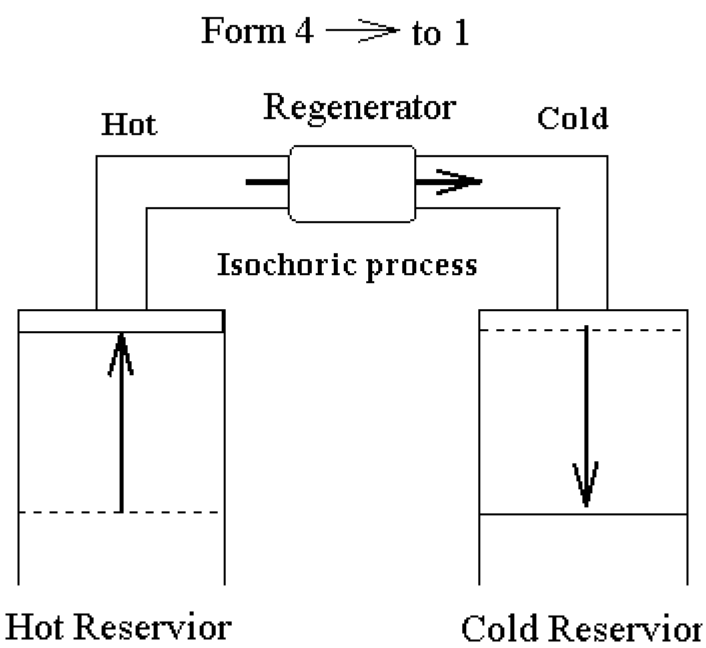
$2 \rightarrow 3$ It is an isochoric process, the left piston moves down while the right piston moves up. The volume of system is kept constant, thus no work has been done by the system, but heat Q_R has been input to the system by the regenerator which causes temperature to raise to θ_H.

Form 3 ⟶ to 4



$3 \rightarrow 4$ It is an isothermal expansion process, the left piston in contact with hot reservoir expanded isothermally at temperature θ_H. Therefore

$$Q_H = P_3 V_3 \ln \frac{V_4}{V_3} = RT_H \ln(r_e)$$



4 → 1 It is an isochoric process which is a reversed process of 2 → 3, but from θ_H to θ_C. The efficiencies of Stirling engine is

$$\eta = 1 - \frac{Q_c}{Q_H} = 1 - \frac{RT_c \ln(r_c)}{RT_H \ln(r_e)} = 1 - \frac{T_c}{T_H}$$

$$\eta_{carnot} = \eta_{stirling}$$

Consider regenerator efficiency η_r

$$Q_H = RT_H \ln(r_c) + (1-\eta_r)(C_v[T_H - T_c])$$

$$Q_c = RT_c \ln(r_c) + (1-\eta_r)(C_v[T_H - T_c])$$

$$\eta_c = \frac{R\ln(r_c)(T_H - T_c)}{RT_c \ln(r_c) + (1-\eta_r)C_v(T_H - T_c)} \quad \text{if } \eta_r = 1$$

$$\eta_{st} = \frac{T_H - T_c}{T_H}$$

1.8 Comparison of Otto, Dual and Diesel cycles

In the previous articles we studied about Otto cycle, diesel cycle and dual cycle and looked at their thermal efficiency. In this article we will take a collective look at these three cycles in order to compare and contrast them, so that we can come to know the relative advantages and disadvantages of these cycles.

1.8.1 Comparison based on same maximum pressure and heat rejection

In this article we will focus on **peak pressure**, **peak temperature** and **heat rejection**. The P-V and T-S diagrams of these three cycles for such a situation are drawn simultaneously as described below.

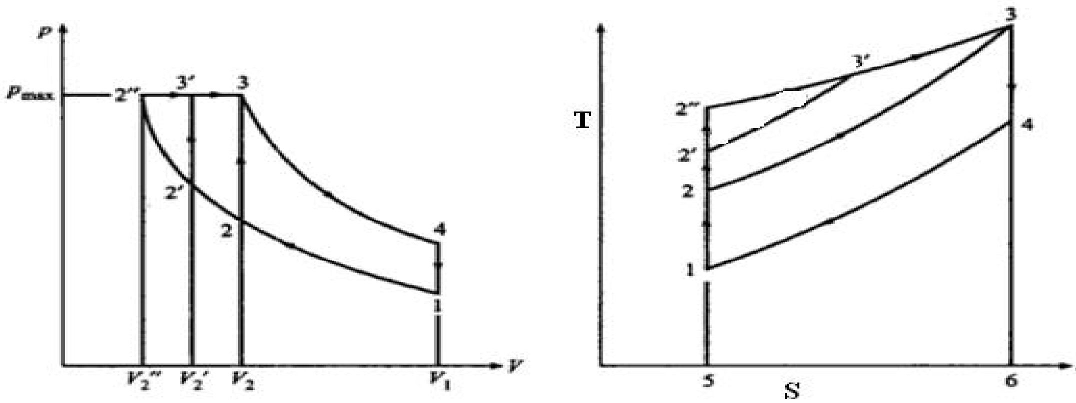


Figure 10 P- V and T- S diagram showing the comparison of Otto, Diesel and Dual cycles

In the above diagrams the following are the cycles

- Otto cycle: 1 – 2 – 3 – 4 – 1
- Dual cycle: 1 – 2’ – 3’ – 3 – 4 – 1
- Diesel cycle: 1 – 2” – 3 – 4 – 1

Remember that we are assuming the same peak pressure denoted by Pmax on the P-V diagram. And from the T-S diagram we know that T3 is the highest of the peak temperature which is again same for all three cycles under consideration. Heat rejection given by the area under 4 – 1 – 5 – 6 in the T-S diagram is also same for each case.

In this case the compression ratio is different for each cycle and can be found by dividing V1 with the respective V2 volumes of each cycle from the P-V diagram. The heat supplied or added in each cycle is given by the areas as follows from the T-S diagram

- Otto cycle: Area under 2 – 3 – 6 – 5 say q1
- Dual cycle: Area under 2’ – 2’ – 3 – 6 - 5 say q2
- Diesel cycle: Area under 2” – 3 – 6 – 5 say q3

It can also be seen from the same diagram that **q3>q2>q1**

We know that thermal efficiency is given by **1 – heat rejected/heat supplied**

Since heat rejected is

Thermal efficiency of these engines under given circumstances is of the following order

Diesel>Dual>Otto

Hence in this case it is the diesel cycle which shows greater thermal efficiency.

1.8.2 Comparison based on same compression ratio and heat rejection

In this article we will focus on constant compression ratio and constant heat rejection. The P-V and T-S diagrams of these three cycles for such a situation are drawn simultaneously as described below.

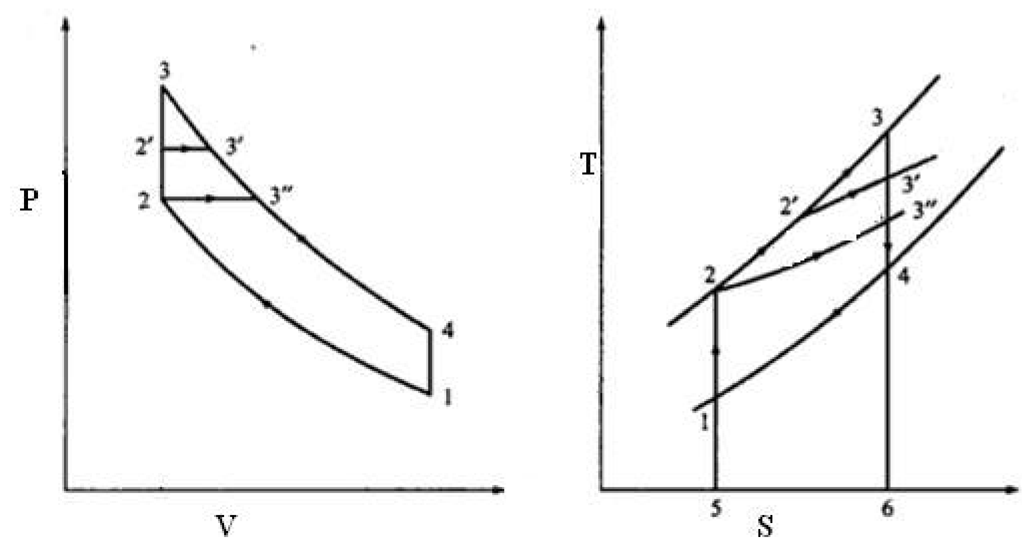


Figure 20 P- V and T- S diagram showing the comparison of Otto, Diesel and Dual cycles

In the above diagrams the following are the cycles

- Otto cycle: 1 – 2 – 3 – 4 – 1
- Dual cycle: 1 – 2 – 2’ – 3’ – 4 – 1
- Diesel cycle: 1 – 2 – 3” – 4 – 1

Remember that we are assuming constant compression ratio for all three cycles which is given by V1/V2

The other parameter which is constant is the heat rejected from the cycle which is given by the following in each case as per the T-S diagram

All cycles: Area under 4 – 1 – 5 – 6 in the T-S diagram

The heat supplied is different in each case and can be established from the T-S diagram as follows

- Otto cycle: Area under 2 – 3 – 6 – 5 say q1
- Dual cycle: Area under 2 – 2’ – 3’ – 6 - 5 say q2
- Diesel cycle: Area under 2 – 3” – 6 – 5 say q3

It can also be seen from the same diagram that **q3<q2<q1**

We know that thermal efficiency is given by **1 – heat rejected/heat supplied**

Since heat rejected is same and we know the order of magnitude of heat supplied, we can combine this information to conclude that

Thermal efficiency

Otto>Dual>Diesel

Hence we see that in this case as well the Otto cycle shows higher thermal efficiency than a dual cycle and even better than the diesel cycle.

MODULE-2

VAPOUR POWER CYCLES

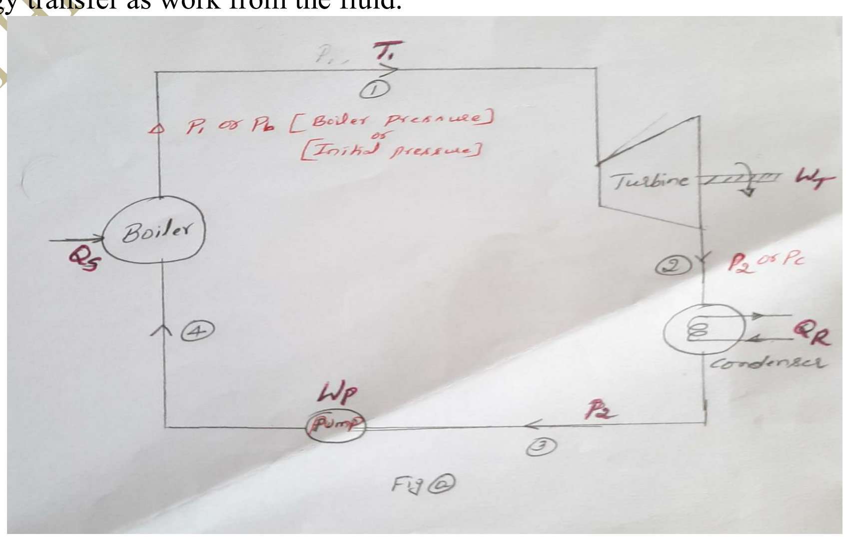
SYLLABUS: Carnot vapour cycles, drawbacks as a reference cycle, simple Rankine cycle, description, T-S diagram, analysis for performance, comparison of Carnot and Rankine cycles, effects of pressure and temperature on Rankine cycle performance. Actual vapour power cycles, Ideal and practical regenerative Rankine cycle, Open and closed feed water heaters, reheat Rankine cycle.

INTRODUCTION: In the vapour power cycle, the phase of working substance changes alternatively. The change of phase allows more energy to be stored in the working substance that can be stored by only sensible heat. The working substance expands as vapour but it is compressed as a liquid with much smaller specific volume, thus a very little of expansion work is used for compression process. The most common working substance is water and thus the cycle and plant are called steam power cycle and steam power plants respectively, even though water is used in the liquid phase during a part of the cycle. Steam power plants generate a major fraction of electric power produced in the world.

A steam power plant consists of following components.

1. Boiler with its mountings and accessories
2. Turbine
3. condenser
4. Feed water pump
5. electric generator
6. chimney
7. draught system
8. Feed water treatment plant

Figure shows a simple steam power plant working on the vapour power cycle. Heat is transferred to the water in the boiler ($Q_1 = Q_{in} = Q_A = Q_S$) from an external source. (Furnace, where fuel is continuously burnt) to raise steam, the high pressure high temperature steam leaving the boiler expands in the turbine to produce shaft work (W_T), the steam leaving the turbine condenses into water in the condenser (where cooling water circulates), rejecting heat (Q_2), and then the water is pumped back (W_P) to the boiler.
Since the fluid is undergoing a cyclic process, the net energy transferred as heat during the cycle must equal the net energy transfer as work from the fluid.



Performance Parameters of vapour power cycle

1. **Thermal Efficiency**: The thermal efficiency of any power plant is the ratio of net work done in the cycle to the heat supplied in the cycle.

$$\boldsymbol{\eta_{th}} = W_{net} / Q_S$$

2. **Back work ratio**: It is the ratio of pump work input to the work developed by the turbine.

$$\mathbf{r_{bw}} = \text{pump work / turbine work} = W_P / W_T$$

3. **Work ratio**: It is the ratio of net work output of the cycle to the work developed by the turbine.

   r_w = net work output of the cycle / turbine work **= $\mathbf{W_{net} / W_T}$ = 1 – back work ratio**

4. **Steam rate**: It is also called specific steam consumption. it relates the power output to the amount of steam necessary to produce it. It is the amount of steam required to produce 1kWhr(3600 kJ) of power. it is denoted by **ssc** and is expressed as and unit is kg/kW-hr

   ssc = (mass of steam in kg/hr) / (power output in kw)

   = **(3600 kJ / kWhr) / ($\mathbf{W_{net}}$ kJ/kg)**

5. **Heat rate** is the amount of heat rejected by a power plant to produce 1kWhr of power. It is expressed in kJ/kWhr.

   Heat rate = 3600/ $\boldsymbol{\eta_{th}}$ kJ/kWhr

CARNOT VAPOUR POWER CYCLE

Carnot vapour power cycle is executed with in the saturation dome of pure substance. It uses two phase fluid as the working medium as shown in figure. Fig (a) gives the arrangement of components in the cycle; fig (b) shows the Carnot power cycle on p-v co-ordinates, fig(c) on T-s co-ordinates.

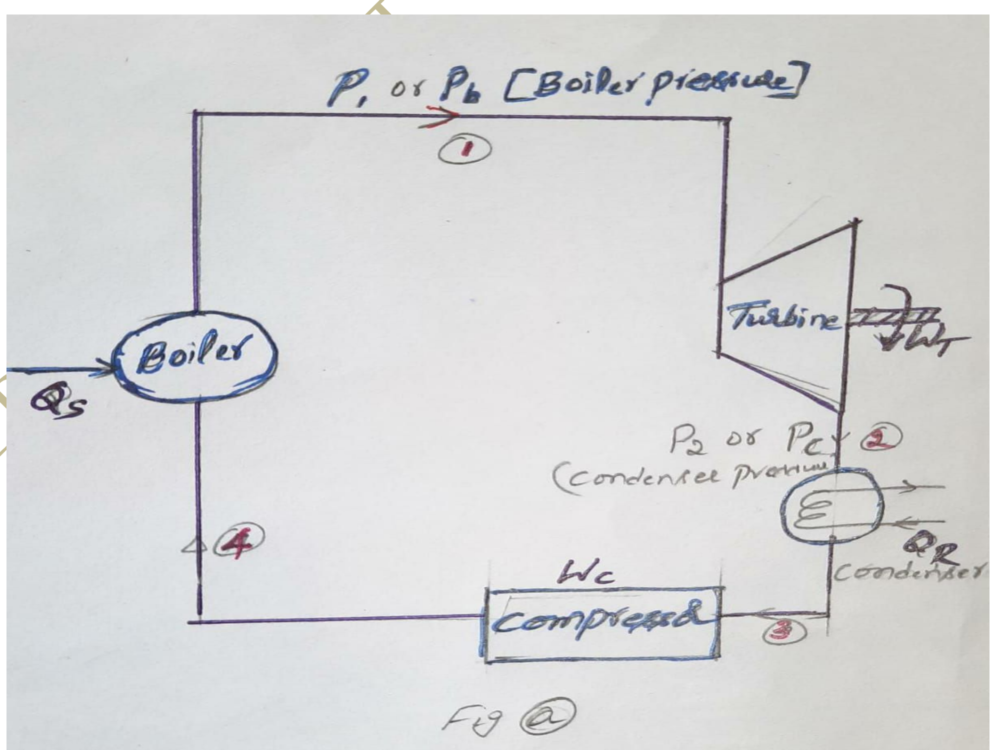
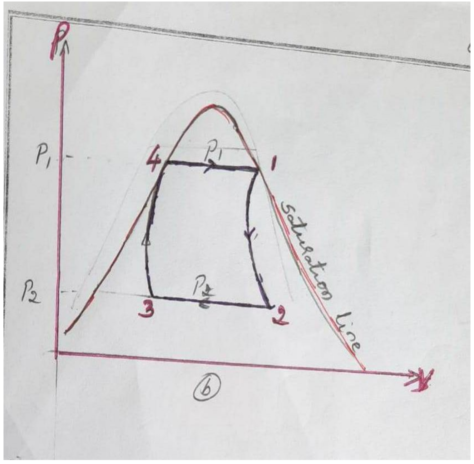
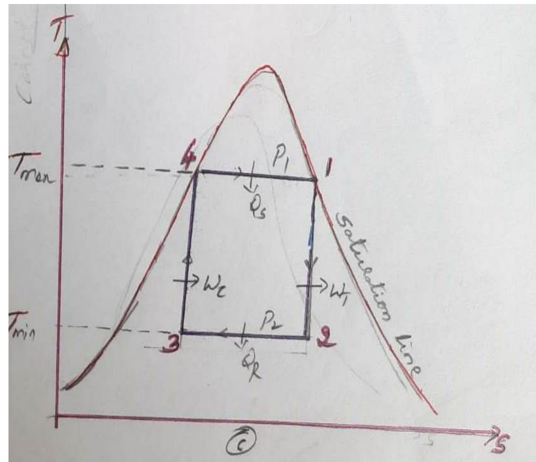


Fig (a)





The four processes in the cycle are follows

1. **Reversible Adiabatic Expansion 1-2:** Saturated steam expands in the turbine. The temperature lowers from T_H or T_{max} to T_L or T_{min}. The state 2 is reached in the wet region.
2. **Controlled condensation 2-3:** During this process, condensation starts from state 2 and stops at state 3 and heat $\mathbf{Q_R}$ per unit mass is rejected in the condenser to the sink at T_{min}.
3. **Reversible Adiabatic Compression 3-4:** The mixture of liquid and vapour is compressed to the saturation liquid state 4 at boiler pressure.
4. **Reversible Isothermal Heat Addition 4-1:** During this process, a quantity of heat $\mathbf{Q_S}$ per unit mass is added in the boiler from the heat sources at the temperature T_{max}.

Analysis of Carnot Vapour Power Cycle

Isothermal heat addition to a vaporising fluid in the boiler; $\mathbf{Q_S} = T_{max}(S_1 - S_4)$

Isothermal heat rejected by the working substance in the condenser; $\mathbf{Q_R} = T_{min}(S_2 - S_3) = T_{min}(S_1 - S_4)$

The net work done of the cycle; $\mathbf{W_{net}} = Q_S - Q_R = (T_{max} - T_{min})(S_1 - S_4)$

The thermal efficiency of the cycle; $\eta_{Carnot} = \mathbf{W_{net} / Q_R}$

$$= (T_{max} - T_{min})(S_1 - S_4) / T_{max}(S_1 - S_4)$$

$$= \mathbf{1 - (T_{max} / T_{min})}$$

The efficiency of the Carnot cycle depends on the operating temperatures and independent of properties of working substance.

Drawbacks or limitations of Carnot Vapour Cycle

1. In the turbine, the dry saturated steam expands isentropically. The quality of steam decreases during expansion. The presence of high moisture content in the steam will lead to erosion and wear of the turbine blade.
2. Isentropic/adiabatic compression process 3-4 involves the compression of liquid-vapour mixture to saturated liquid state. There are 3 practical difficulties associated with this process:

   a) Control of condensation at state 3, before reaching to saturated liquid state

b) Compression of two phase (water+ steam) system. Because of large specific volume of vapour than liquid, the compressor size and work input will have to be large.
c) Higher compression work will reduce the thermal efficiency of the plant.

Note: Many practical difficulties associated with the Carnot vapour cycle are eliminated in Rankine cycle. The steam coming out of the boiler is usually in superheated state, and expands in the turbine. After expanding in the turbine, the steam is condensed completely in the condenser. So the Rankine cycle is an ideal vapour cycle and is used in steam power plants.

RANKINE CYCLE

The Rankine cycle is an ideal reversible cycle and used in steam power plants. Figure (a) shows the Rankine cycle and fig(b) on (T-S) and fig (c) on (h-s) coordinates. The liquid, vapour and wet vapour regions are also indicated with the help of saturation curve.

In fig b & c, the cycle 1-2-3-4-1 represents an ideal Rankine cycle using saturated steam and cycle 1'-2'-3-4-1 represents an ideal Rankine cycle with Superheated steam at the turbine entry. The Rankine Cycle consists of following four internally reversible processes:

Process1-2: Isentropic expansion of the working fluid in the turbine from the boiler pressure to condenser pressure.

Process 2-3: Heat rejection from the working fluid at constant pressure in the condenser till the fluid reaches the saturated liquid state3.

Process 3-4: Isentropic compression of the working fluid in the pump to the boiler pressure at the state 4 in the compressed liquid region.

Process 4-1: Heat addition to working fluid at constant pressure in the boiler from state 4 to 1.

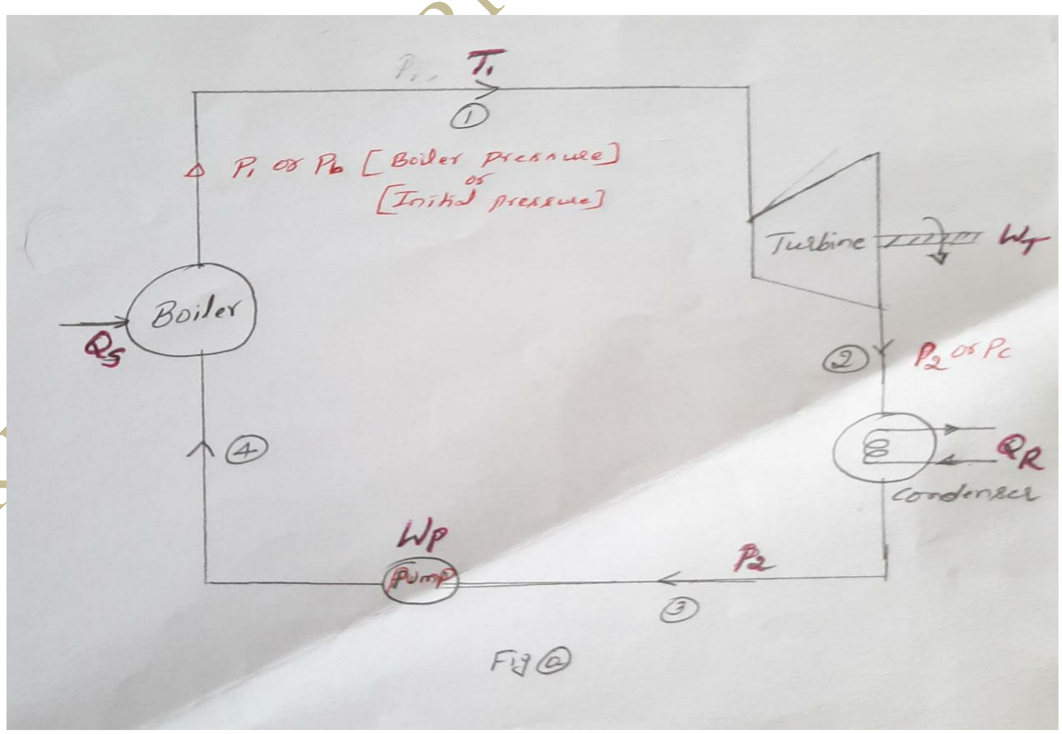
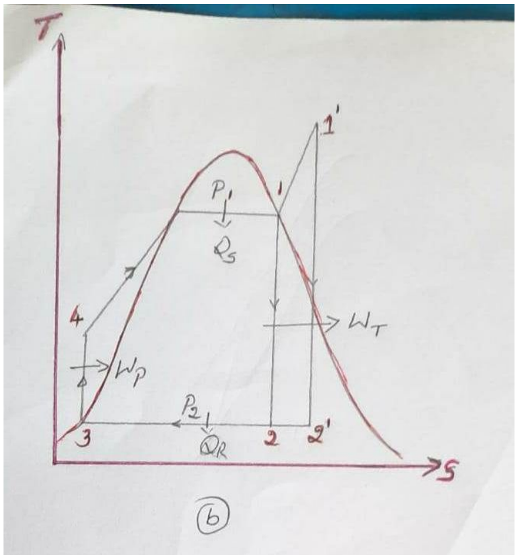
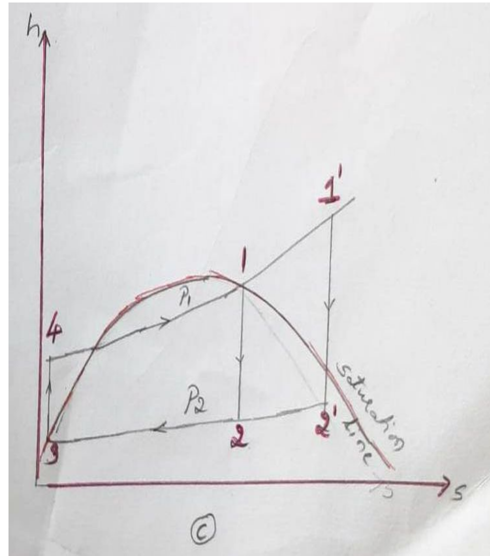


Fig a





Analysis of Rankine cycle:

Assume 1kg flow of steam in the cycle. Applying Steady flow energy equation (S.F.E.E) to each component in the cycle. If the K.E & P.E are neglected then the S.F.E.E reduce to Q-**W= Δh**

(i) For constant pressure heat addition process 4-1 in the boiler (W=0);

$$Q_{1\text{-}4} = \mathbf{Q_S} = h_1\text{-}h_4$$

(ii) For isentropic expansion process 1-2 in the turbine (Q=0); $\mathbf{W_T} = h_1\text{-}h_2$

(iii) For constant pressure heat removal process 2-3 in the condenser (W=0);

$$Q_{2\text{-}3} = \mathbf{Q_R} = h_2\text{-}h_3$$

(iv) For isentropic compression 3-4 in the pump (Q=0); $\mathbf{W_P} = h_4\text{-}h_3$

Where h_3 is h_f enthalpy of liquid at condenser pressure P_2, h_4 is the enthalpy of the water at state 4.

Then the isentropic compression work is W_P is obtained as $\mathbf{W_P = \mathcal{V}_f (P_1 - P_2) \times 100}$ **kJ/kg**

Where $\mathcal{V}_f$ is the specific volume of the liquid at condenser pressure P_2.
(Where P_1& P_2 are in kPa or kN/m^2 for only pump work to balance the equation to get kJ/kg)

Thermal efficiency of power cycle is given by
η = net work done in the cycle / heat supplied in the cycle = $\mathbf{W_{net}} \;/\; \mathbf{Q_S}$
$\mathbf{W_{net}} = W_T - W_P = (h_1\text{-}h_2) - (h_4\text{-}h_3)$
Also, $\mathbf{W_{net}} = Q_S - Q_R = (h_1\text{-}h_4) - (h_2\text{-}h_3) = (h_1\text{-}h_2) - (h_4\text{-}h_3)$ kJ/kg
Thus efficiency of Rankine cycle can be expressed as

$$\boldsymbol{\eta}_{\mathbf{Rankine}} = (Q_S - Q_R) / Q_S = 1 - Q_S / Q_R = 1 - [(h_2\text{-}h_3)/(h_1\text{-}h_4)]$$

Note: Mean temperature of heat addition

T_{mean}= Heat supplied / Change in Entropy at heat supplied = $Q_S / (S_1\text{-}S_4)$ °K

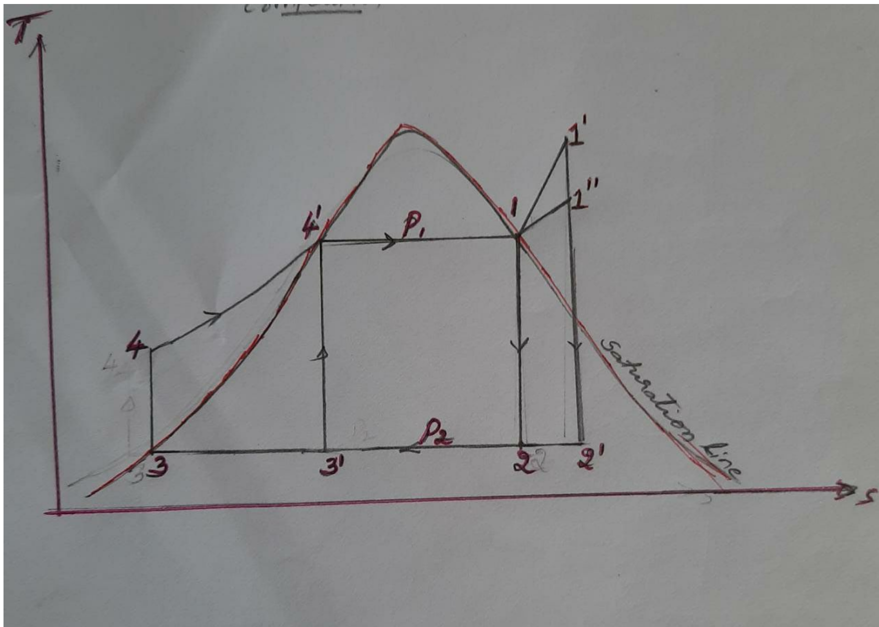
Relative Efficiency or Efficiency ratio

It is defined as the ratio of actual thermal efficiency of steam power plant to the corresponding Rankine efficiency. It is denoted by $\eta_{relative}$ expressed as

$$\eta_{relative} = \text{Actual thermal efficiency / Rankine efficiency}$$

COMPARISON BETWEEN CARNOT & RANKINE CYCLES

Figure shows the graphical comparison b/w Carnot and Rankine cycles with p-v and T-s diagrams



Cycle 1-2-3’-4’-1 Carnot cycle with saturated steam

Cycle 1”-2’-3’-4’-1 Carnot cycle with superheated steam

Cycle 1-2-3-4-1 Rankine cycle with saturated steam

Cycle 1’-2’-3-4-1 Rankine cycle with superheated steam

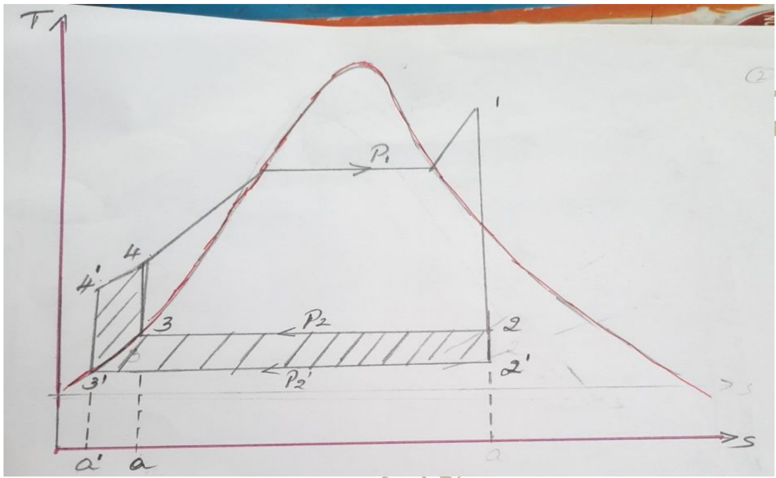
1. In the Rankine cycle, liquid water is pumped during the process 3-4. Since the specific volume of the working substance after complete condensation at the state 3 becomes very small, therefore, the back work ratio in the Rankine cycle is almost negligible. While the specific volume of the liquid and mixture at the state 3’ is large, thus a large compression work is required in Carnot vapour power cycle.
2. Turbine work in both the cycles is same.
3. There are higher rates of heat transfer in the boiler and condenser due to long processes. Consequently, the Rankine cycle requires a lower steam rate than Carnot cycle. Therefore, the plant size of the Rankine cycle is smaller.
4. The Rankine cycle uses a part of heat supplied at constant temperature. Therefore, its efficiency is lower than that of steam of Carnot cycle.

EFFECT OF OPERATING VARIABLES ON RANKINE CYCLE (Methods to increase in thermal efficiency of Rankine cycle)

1. **Effect of Exhaust Pressure and Temperature:**

   Figure shows effect of condenser pressure on the Rankine cycle. The steam enters the condenser as a saturated mixture of vapour and moisture at the turbine back pressure P_2. If this pressure is lowered, the saturation temperature of exhausted steam decreases and amount of heat rejection in the condenser also decreases. The efficiency of Rankine cycle increases by lowering the exhaust pressure.

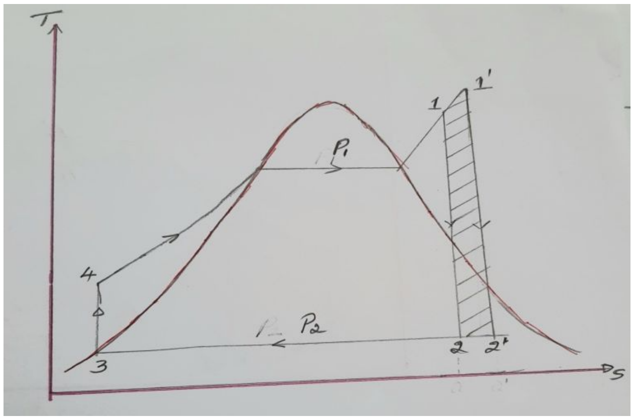
   As the turbine pressure P_2 decreases to $\mathbf{P_2}$' the heat rejection decreases by an area 2-3-3'-2'-2. The heat transfer to steam is also increased by an area a'-4'-4-a-a'.Thus net work done and efficiency of the cycle increases.



Limitations: Lowering the back pressure causes increases in moisture content of the steam leaving the turbine. It is an unfavourable factor, because if the moisture content of steam in low-pressure stages of the turbine exceeds 10%, there is a decrease in turbine efficiency and erosion of turbine blade may also be a serious problem.

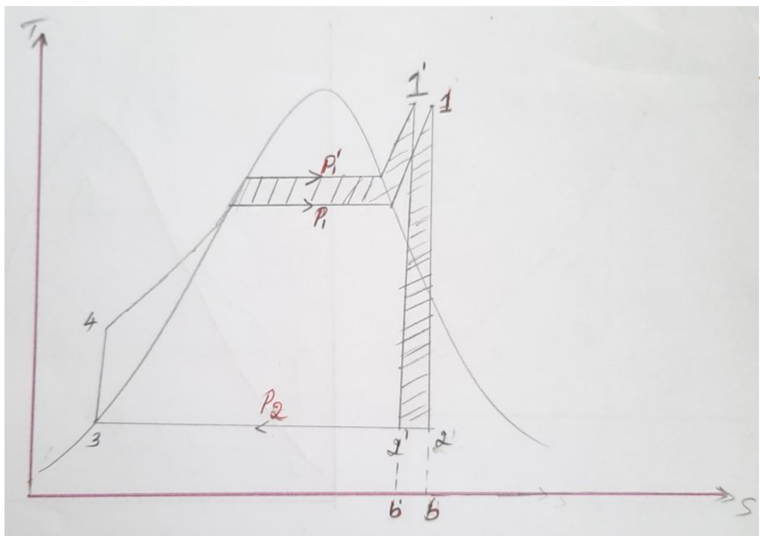
2. **Effect of superheating.**

   Superheating of steam increases the mean temperature of heat addition. The effect of superheated steam on the performance of the Rankine cycle is as shown in the figure. The increase in superheat is shown by the line 1-1'. The area 1-1'-2'-2-1 represents increases in net work done during the cycle. The area under the curve 1-1' represents increase in heat input. Therefore, the thermal efficiency of the Rankine cycle increases and it is observed that the specific steam consumption decreases as the steam is superheated.



Superheating of steam to higher temperature is desirable, because the moisture content of the steam leaving the turbine decreases as indicated by the state 2 as shown in fig. However, the metallurgical considerations, usually the maximum temperature of steam are limited to 650°C.

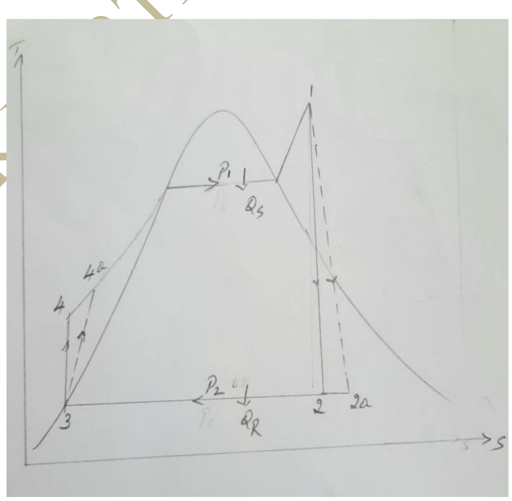
3. **Effect of increase in boiler pressure**
   By increasing the boiler pressure, the mean temperature of heat addition increases, and thus raises the thermal efficiency. Figure shows the effect of boiler pressure on Rankine cycle efficiency. By keeping the maximum temperature T_{max} and condenser pressure P_2 constant if boiler pressure increases, the heat rejection decreases by an area b'-2'-2-b-b'. <u>The net work done by the cycle efficiency increases, with increases in maximum pressure.</u>

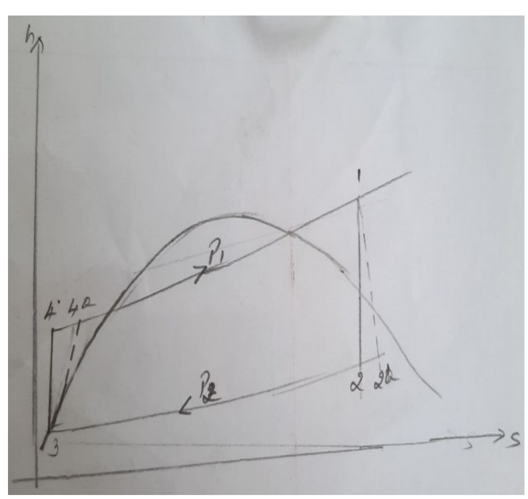


We conclude that the efficiency of Rankine cycle can be increased by lowering the condenser pressure, by increasing the boiler pressure and by superheating steam.

ACTUAL VAPOUR POWER CYCLE
Rankine cycle considered as an ideal vapour power cycle. In actual practice, the cycle involves irreversibility and losses and therefore, the efficiency of actual vapour cycle is less than that of ideal Rankine cycle. The losses associated with actual processes are as shown in figure.

Ideal cycle 1-2-3-4-1, actual cycle 1-2a-3-4a-1





Turbine Losses: Actual expansion process **1-2a** in the turbine is an irreversible process as shown in figure. The entropy increases during the actual expansion process. Some heat may also transfer to the surroundings. Thus the actual work corresponding to isentropic process 1-2. The isentropic efficiency of the turbine is given as

η_T = Actual turbine work / Isentropic or ideal turbine work = $\mathbf{(h_1 - h_{2a}) / (h_1 - h_2)}$

Pump losses: The pump losses are very similar to turbine losses. The actual compression process **3-4a** in the pump is an irreversible process. The entropy increases during the actual compression process. Therefore, the actual work corresponding to the isentropic process 3-4. The isentropic efficiency of the pump is given as,

η_p = Isentropic or ideal pump work / Actual pump work $= \mathbf{(h_4 - h_3) / (h_{4a} - h_3)}$

Piping losses: When the working fluid passes through the tubes of the boiler and condenser, the pressure drops due to frictional effects and transfer of heat to surroundings. Both pressure drop and heat transfer decreases the availability of the steam entering the turbine. However these effects are negligible and not shown in the figure.

REHEATING OF STEAM

If the steam expands completely in a single stage then steam coming out the turbine is very wet. The wet steam carries suspended moisture particles, which are heavier than the vapour particles, thus deposited on the blades and causing its erosion. In order to increase the life of the turbine blades, it is necessary to keep the steam dry during its expansion. It is done by allowing the steam to expand to an intermediate pressure in a high-pressure turbine, and then taking it out and sending back to the boiler, where it is out and sending back to the boiler, where it is reached at constant pressure, until it reaches the inlet temperature of the first stage. This process is called reheating during which heat is added to the steam. The reheated steam then further expands in the next stage of the turbine. Due to reheating, the work output of the turbine increases, thus improving the thermal efficiency and also it reduces the steam rate per kWh. The main disadvantage of the reheat cycle is to increase the cost and size of the plant.

REHEAT RANKINE CYCLE

The reheat cycle is designed to take advantage of higher boiler pressure by eliminating the problem of excessive moisture content in the exhaust steam. In a reheat Rankine cycle; the steam is expanded in number of stages. After each stage of expansion, steam is reheated in the boiler. Then, it expands in the next stage of turbine and is finally exhausted to the condenser.

Figure (a) shows the reheating process and fig (b) and (c) shows the reheating process 2-3 on T-s and h-s diagrams respectively. The steam at state 1 enters in the first stage of turbine and expands isentropically to the state2. The quality of steam at the state 2, is either just dry or slightly wet and thus it is taken back in boiler and is reheated to original superheated temperature $T_3 = T_1$ at constant pressure P_2. Then this turbine and expands to back pressure at the state 4.

The amount of heat added during reheating $Q_{reheat} = (h_3 - h_2)$ kJ/kg

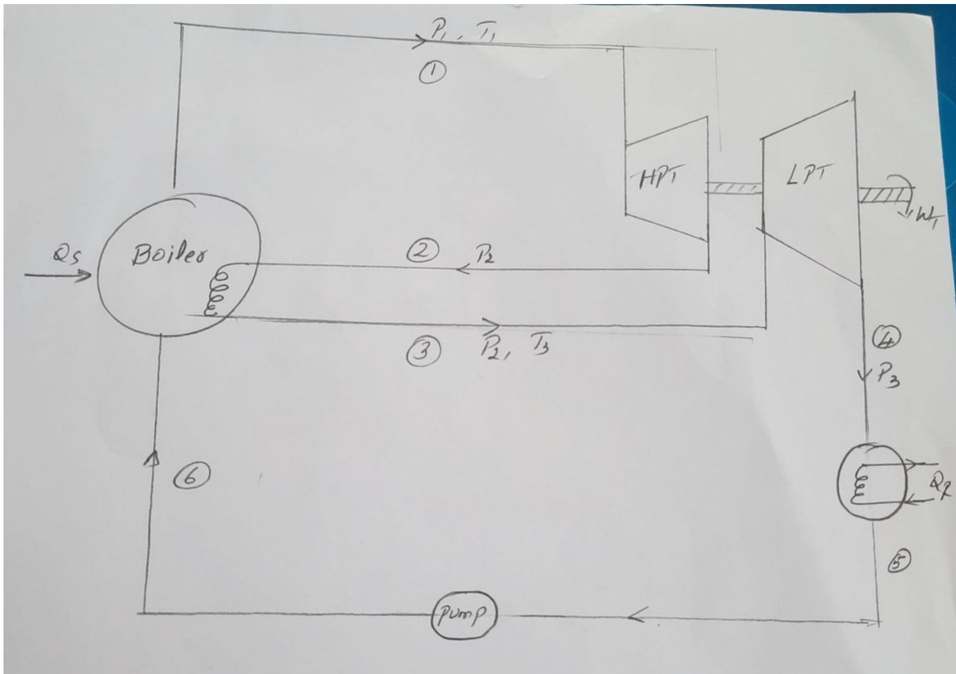
The total heat supplied per kg in two stages $Q_S = (h_1 - h_6) + (h_3 - h_2)$ kJ/kg

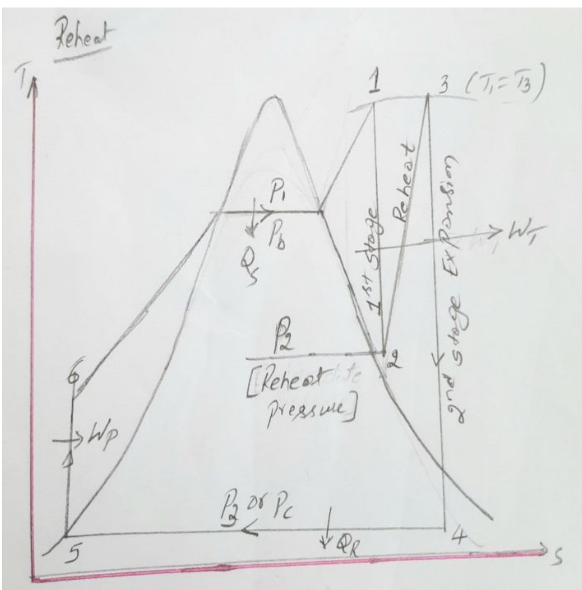
For isentropic expansion in two stages, the total work done per kg of steam

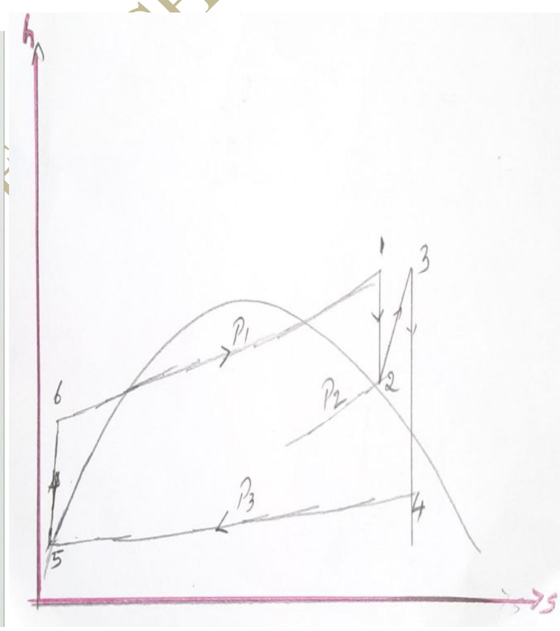
$$W_T = HPT + LPT = [(h_1 - h_2) + (h_3 - h_4)] \text{ kJ/kg}$$

The pump work per kg of steam $W_P = (h_6 - h_5) = v_f \mathbf{(P_1 - P_3)} \times 100$ kJ/kg

h_5 is the h_f enthalpy of the liquid at condenser pressure P_3 , v_f at condenser pressure $\mathbf{P_3}$.







The net work done per kg of steam; $W_{net} = W_T - W_P$

$$= [(h_1-h_2) + (h_3 - h_4) - (h_6-h_5)] \text{ kJ/kg}$$

However the pump work is very small in comparison with the turbine work, thus it is neglected in most cases.

The heat rejected in the condenser per kg of steam $Q_R = (h_4-h_5)$ kJ/kg

The efficiency of the turbine with reheating is given by

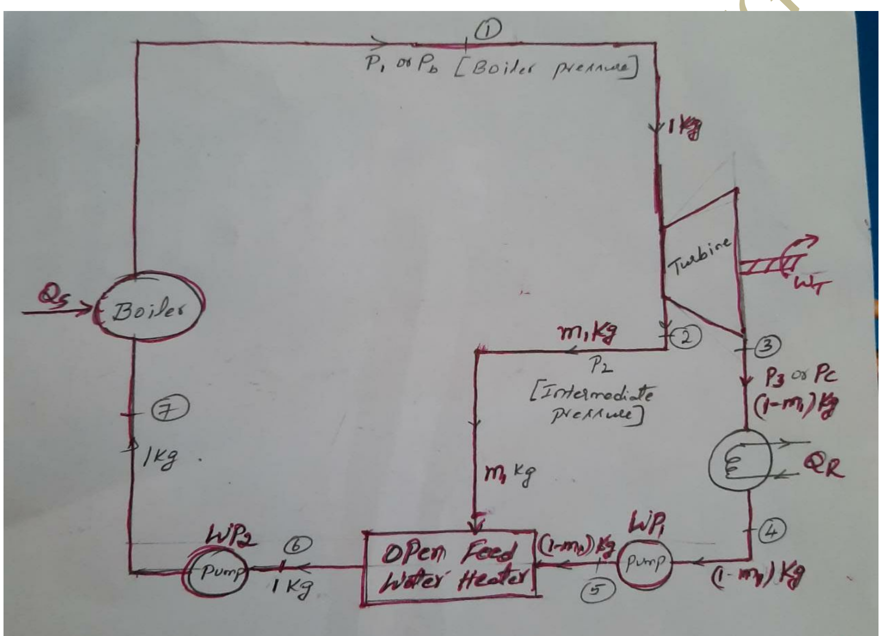
$$\eta_{reheat} = W_{net} / Q_S = (Q_S - Q_R) / Q_S = 1 - (Q_R / Q_S)$$

$$= 1 - \{(h_4-h_5) / (h_1 - h_6) + (h_3 - h_2)\}$$

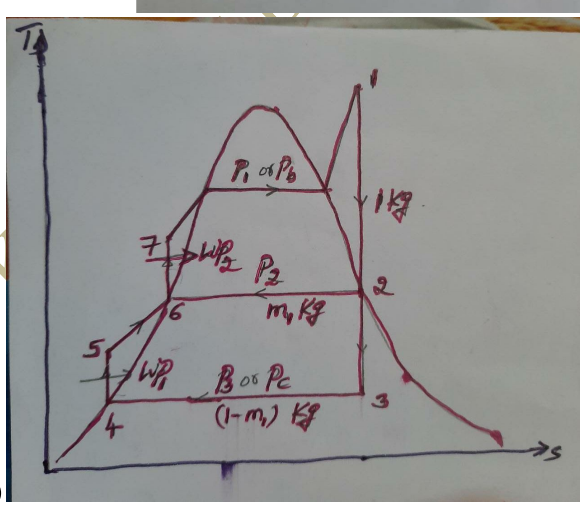
IDEAL REGENERATIVE CYCLE

REGENERATION WITH OPEN or CONTACT TYPE FEED-WATER HEATER

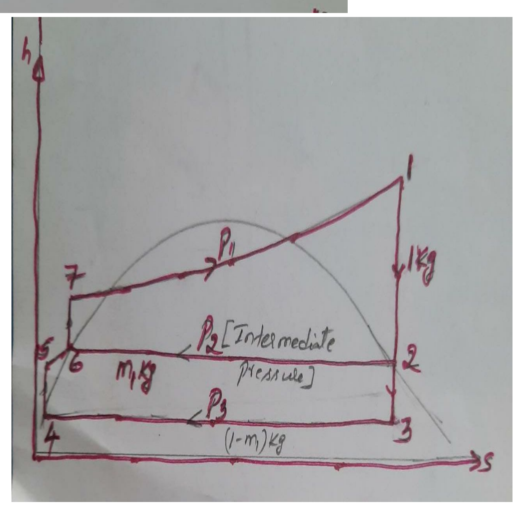
A regenerative cycle with open feed water heater is shown in figure (a). A part of superheated stem which can enters the turbine at state 1, is extracted from the turbine at the intermediate state 2 of turbine expansion process. The extracted steam is supplied to a heat exchanger known as feed water heater. The remaining amount of steam in the turbine expands completely to condenser pressure at state 3. The condensate, a saturate liquid at state 4 is pumped isentropically by low pressure pump to pressure extracted steam. The compressed liquid at state 5 enters the feed water heater and it mixes with steam extracted from the turbine. Due to direct mixing process, the feed water heater is called <u>open or direct contact type feed water heater.</u> The portion of steam extracted is so adjusted to make the mixture leaving the feed water to be saturated at state 6. Now this saturated water is pumped by high pressure pump to boiler pressure sate 7. With the regeneration, the average temperature at which is supplied has been increased; therefore, Rankine cycle efficiency improves. fig(a)



(b)



(c)



Analysis:

Let 1 kg of steam be leaving the boiler and entering the turbine.

m_1 kg of steam per kg, is extracted at the state 2 from the turbine at intermediate pressure P_2.

$(1\text{-}m_1)$ kg of steam per kg flow through the remaining part of the turbine during expansion from 2-3, condensation from 3-4 and pumping from 4-5.

$(1\text{-}m_1)$ kg of steam enters in open feed water heater and mix with m_1 kg of steam blown from the turbine at the state 2. After mixing, the mass of saturated liquid becomes 1kg at the state 6 and it is pumped to boiler pressure at the state 7.

Applying S.F.E.E to mixing process 2-6; $(1\text{-}m_1)\, h_5 + m_1 h_2 = h_6$

$$m_1 = (h_6 - h_5) / (h_2 - h_5) \text{ kg}$$

The heat supplied in the boiler $Q_S = (h_1 - h_7)$ kJ/kg

Heat rejected in condenser $Q_R = (1\text{-}m_1)\,(h_3 - h_4)$ kJ/kg

Turbine work $W_T = (h_1 - h_2) + (1\text{-}m_1)\,(h_2 - h_3)$ kJ/kg

Pump work $W_P = HPP + LPP = (h_7 - h_6) + (1\text{-}m_1)\,(h_5 - h_4)$ kJ/kg

$HPP = W_{P1} = V_f\,(P_2 - P_3) \times 100$ kJ/kg where V_f at P_3)

$LPP = W_{P2} = V_f\,(P_1 - P_2) \times 100$ kJ/kg where V_f at P_2

Net work done per kg of steam,

$$\mathbf{W_{net}} = W_T - W_P$$

$$= [(h_1 - h_2) + (1\text{-}m_1)\,(h_2 - h_3)] - [(h_7 - h_6) + (1\text{-}m_1)\,(h_5 - h_4)]$$

Thermal efficiency of regenerative cycle;

$$\boldsymbol{\eta_{reg}} = 1 - (Q_R / Q_S) = 1 - [(1\text{-}m_1)\,(h_3 - h_4) / (h_1 - h_7)]$$

Advantages of regeneration

1. It raises the temperature of feed water to saturation temperature, and thus the amount of heat is addition in the boiler reduces.
2. The heat is added in the boiler at a higher average temperature.
3. Open feed water serves as dearator to remove the air and other non-condensable gases from the feed water, otherwise they would causes corrosion.

Note: In the regenerative cycle, feed water enters the boiler at temperature T_7 and its mean temperature of heat addition is

$$T_{m,reg} = Q_S / (S_1 - S_7) = (h_1 - h_7) / (S_1 - S_7)\ \text{°K}$$

The mean temperature of heat addition without regeneration for simple Rankine cycle operating b/w same pressure P_1 and P_3 would be

$$T_{m,Rankine} = Q_S / (S_1 - S_5) = (h_1 - h_7) / (S_1 - S_5) \ \ °K$$

The efficiency of regenerative of regenerative cycle will be more than Rankine cycle because of the mean effective temperature during heat addition. since $\mathbf{T_{m,reg} > T_{m,Rankine}}$

Numerical:

(Before solving the numerical pressure should be converted to bar if it is given in kpa or Mpa).

1. A simple Rankine cycle works between the boiler pressure of 25bar and condenser pressure of 0.2bar. The steam is dry saturated before the throttling in the turbine. Determine, quality of steam at the end of expansion, Rankine cycle efficiency, work ratio, specific steam consumption,. turbine shaft work, heat loss in the condenser if steam flow rate is 10 kg/sec.

2. A steam turbine receives steam at 15 bar and 300 °C and leaves the turbine at 0.1 bar and 4% moisture. Determine , Rankine efficiency, steam consumption.

3. A turbine is supplied with steam at a pressure of 30 bar and a temp of 400°C.The steam then expands isentropically to a pressure of 0.08 bar. Find the dryness fraction at the end of expansion and thermal efficiency of the cycle. If the steam is reheated at 5.5 bar to a temp of 400°C and then expanded isentropically to a pressure of 0.08 bar, what will be the dryness fraction and thermal efficiency of the cycle.

4. An ideal Rankine cycle with reheat is designed to operate according to the following specification: Pressure at the inlet of HP turbine = 20MPa

Temp of steam at the inlet of HP turbine = 550°C
Temp of steam at the end of reheat = 550°C
Pressure of steam at the turbine exit =15kPa
Quality of steam at the turbine exit = 90%.
Determine i) Reheat pressure ii) Temp in the condenser
iii) Ratio of pump work to turbine work iv) cycle thermal efficiency

5. A steam power station uses the following cycle; steam at the boiler outlet is 150 bar, 550°C, Reheat at 40 bar, 550°C, condenser at 0.1 bar. Using Mollier chart and assuming all the processes are ideal, find

i) Quality of steam ii) Cycle efficiency iii) Steam rate

6. Steam at 20 bar, 360°C is expanded in a steam turbine to 0.08bar. It is then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler.

(a) Assuming ideal processes, find per kg of steam the net work and cycle efficiency. (b) If the turbine and pump have each 80% efficiency, find the % reduction in the net work and cycle efficiency.

7. In a single regenerative cycle the steam enters the turbine at 30bar, 400°C and exhaust pressure is 0.1 bar. The feed water heater is a direct contact type which operates at 5 bar.ion Find (a) the efficiency and steam rate of the cycle (b) the increase in mean temp of heat addition, efficiency and steam rate, as compared the Rankine cycle without regeneration. Neglect pump work.

8. A steam power plant operates on an ideal reheat Rankine cycle b/w the pressure limits of 9MPa and 10kPa. The mass flow rate of steam through the cycle is 25 kg/s. Steam enters both the stages of the turbine at 500°C. If the moisture content of the steam exiting the low-pressure turbine should not to

exceed 10%; determine, The reheat pressure ii) total rate of heat input in the boiler iii) thermal efficiency

9. An ideal regenerative steam cycle operates with the steam entering the turbine at 30 bar and 500°C and is exhausted at 0.1 bar. A feed water heater is used, which operates at 5 bar. calculate

i) the thermal efficiency ii) steam rate of the cycle iii) increase in avg temp of heat addition, efficiency, and steam as compare to an ideal Rankine cycle operates b/w same conditions.

10. In a single heater regenerative cycle the steam enters the turbine at 30bar, 400°C and the exhaust pressure is 0.1 bar. The feed water heater is a direct contact type which operates at 5 bar. Find:

i) The efficiency and the steam rate of the cycle
ii) The increase in mean temp of heat addition, efficiency & steam rate as compared to the Rankine cycle with out regeneration. Pump work may be neglected.

11. A regenerative Rankine cycle includes open feed water heater. The steam enters the turbine at 4 Mpa and 400°C. The steam expands up to 400 kPa and then some quality of steam is extracted from the turbine and is supplied to feed water heater. The water leaves the feed water as saturated liquid at 400 kPa. The remaining steam completes its expansion to 10 kPa. Calculate the cycle efficiency and steam rate.

MODULE-3

Testing of I.C.Engines

Introduction: - The basic task in the design and development of I.C.Engines is to reduce the cost of production and improve the efficiency and power output. In order to achieve the above task, the engineer has to compare the engine developed by him with other engines in terms of its output and efficiency. Hence he has to test the engine and make measurements of relevant parameters that reflect the performance of the engine. In general the nature and number of tests to be carried out depend on a large number of factors. In this chapter only certain basic as well as important measurements and tests are described.

Important Performance Parameters of I.C.Engines:- The important performance parameters of I.C. engines are as follows:

(i) Friction Power (motoring power),

(ii) Indicated Power,

(iii) Brake Power,

(iv) Specific Fuel Consumption,

(v) Air – Fuel ratio

(vi) Thermal Efficiency

(vii) Mechanical Efficiency.

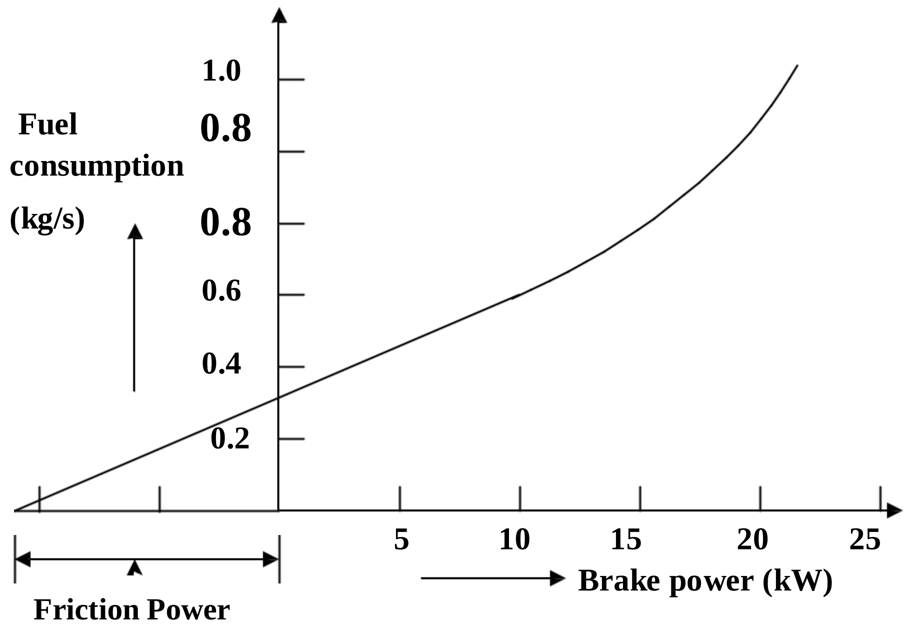
Measurement of Frictional Power (F.P)

The various methods are used to determine frictional power are

a. Willan's line method
b. From the measurement of indicated and brake power
c. Morse test
d. Motoring test.

Measurement of Friction Power:- Friction power includes the frictional losses and the pumping losses. During suction and exhaust strokes the piston must move against a gaseous pressure and power required to do this is called the "pumping losses". The friction loss is made up of the energy loss due to friction between the piston and cylinder walls, piston rings and cylinder walls, and between the crank shaft and camshaft and their bearings, as well as by the loss incurred by driving the essential accessories, such as water pump, ignition unit etc.

Willan's Line Method:- This method is also known as fuel rate extrapolation method. In this method a graph of fuel consumption (vertical axis) versus brake power (horizontal axis) is drawn and it is extrapolated on the negative axis of brake power. The intercept of the negative axis is taken as the friction power of the engine at that speed.



As shown in the figure, in most of the power range the relation between the fuel consumption and brake power is linear when speed of the engine is held constant and this permits extrapolation. Further when the engine does not develop power, i.e. brake power = 0, it consumes a certain amount of fuel. This energy in the fuel would have been spent in overcoming the friction. Hence the extrapolated negative intercept of the horizontal axis will be the work representing the combined losses due to friction, pumping and as a whole is termed as the frictional loss of the engine. This method of measuring friction power will hold good only for a particular speed and is applicable mainly for compression ignition engines.

The main draw back of this method is the long distance to be extrapolated from data between 5 and 40 % load towards the zero line of the fuel input. The directional margin of error is rather wide because the graph is not exactly linear.

From the Measurement of Indicated Power and Brake Power:- This is an ideal method by which friction power is obtained by computing the difference between the indicated power and brake power. The indicated power is obtained from an indicator diagram and brake power is obtained by a brake dynamometer. This method requires elaborate equipment to obtain accurate indicator diagrams at high speeds.

$$FP = IP - BP$$

Morse Test:- This method can be used only for multi – cylinder IC engines. The Morse test consists of obtaining indicated power of the engine without any elaborate equipment. The test consists of making, in turn, each cylinder of the engine inoperative and noting the reduction in brake power developed. In a petrol engine (gasoline engine), each cylinder is rendered inoperative by *"shorting"* the spark plug of the cylinder to be made inoperative. In a Diesel engine, a particular cylinder is made inoperative by cutting off the supply of fuel. It is assumed that pumping and friction are the same when the cylinder is inoperative as well as during firing.

In this test, the engine is first run at the required speed and the brake power is measured. Next, one cylinder is cut off by short circuiting the spark plug if it is a petrol engine or by cutting of the fuel supply if it is a diesel engine. Since one of the cylinders is cut of from producing power, the speed of the engine will change. The engine speed is brought to its original value by reducing the load on the engine. This will ensure that the frictional power is the same.

Assuming a four-cylinder engine,

Let IP_1, IP_2, IP_3, IP_4 be the Indicated power of each cylinders 1,2,3&4 respectively.

FP_1, FP_2, FP_3, FP_4 be the friction power of each cylinders.

When all the cylinders are working,

Total brake power $BP = BP_1 + BP_2 + BP_3 + BP_4$

$$BP = (IP_1 - FP_1) + (IP_2 - FP_2) + (IP_3 - FP_3) + (IP_4 - FP_4)$$

$$BP = (IP_1+IP_2+IP_3+IP_4) – (FP_1-FP_2 -FP_3- FP_4) \text{ ----------------------(1)}$$

When the first cylinder is cut out, then BP of first cylinder is

$$BP_1= (IP_2+IP_3+IP_4) – (FP_1-FP_2 -FP_3- FP_4) \text{ ---------------------------(2)}$$

subtracting (2) from (1)

$$IP_1= BP-BP_1$$

When the second cylinder is cut out, the BP of the second cylinder is

$$BP_2= (IP_!+IP_3+IP_4) – (FP_1-FP_2 -FP_3- FP_4) \text{ ---------------------------(3)}$$

subtracting (3) from (1)

$$IP_2= BP-BP_2$$

similarly , when third cylinder is cut out,

$$IP_3= BP-BP_3$$

When fourth cylinder cut out,

$$IP_4=BP-BP_4$$

Hence total indicated power of the engine is

$$IP= (IP_1+IP_2+IP_3+IP_4)$$

$$IP = (BP-BP_1) +(BP-BP_2) + (BP-BP_3) + (BP-BP_4)$$

In general, IP for n cylinders is given by,

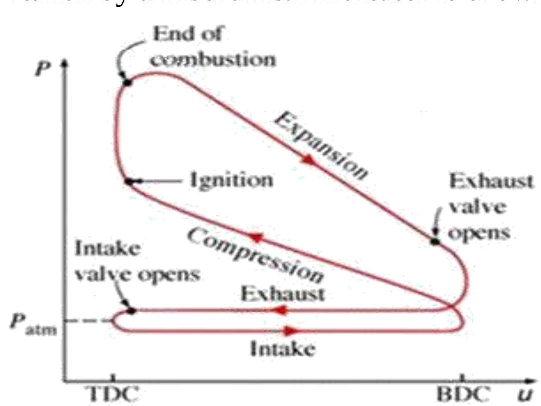
$$IP_n = BP_n + FP_n$$

MEASUREMENT OF INDICATED POWER

The power developed inside the engine cylinder is known as Indicated Horse Power and is designated as IP.

The IP of an engine at a particular running condition is obtained from the indicator diagram. The indicator diagram is the *p-v* diagram for one cycle at that load drawn with the help of indicator fitted on the engine. The construction and use of mechanical indicator for obtaining *p-v* diagram is already explained.

A typical *p-v* diagram taken by a mechanical indicator is shown in Figure .



The areas, the positive loop and negative loop, are measured with the help of a planimeter. The height multiplied by spring-strength (or spring number) gives the indicated mean effective pressure of the cycle.

Where *S* is spring scale and it is defined as a force per unit area required to compress the spring through a height of one centimetre ($N/m^2/cm$) or bar/cm.

Generally the area of negative loop is negligible compared with the positive loop and it cannot be easily measured especially when it is taken with the spring used for taking positive loop. Special light springs are used to obtain the negative loop. When two different springs are used for taking the *p-v* diagram of positive and negative loop, then the net indicated mean effective pressure is given by

$$P_{mi} = S*a / l \quad \text{bar}$$

Where S = Spring strength used for taking *p-v* diagram bar/cm
a= Area in indicator diagram in cm^2
l= *Length of indicator diagram in cm*

The IP developed by the engine is given by

$$\mathbf{IP=(10/6)\ knP_{mi}\ LAN} \quad \textbf{in kW}$$

Where,
P_{mi}= Indicated mean effective pressure in bar.
L = stroke or stroke length in m.
A = Area of cylinder in m^2
N = Speed of the engine in rpm.
K = simplification factor =1 for 2stroke engine
= ½ for 4stroke engine.
n= number of cylinders.

MEASUREMENT OF B.P

Part of the power developed in the engine cylinder is used to overcome the internal friction. The net power available at the shaft is known as brake power and it is denoted by B.P. The arrangement used for measuring the BP of the engine is described below:

(a) Prony Brake. The arrangement of the braking system is shown in Figure . It consists of brake shoes made of wood and these are clamped on to the rim of the brake wheel by means of the bolts. The pressure on the rim is adjusted with the help of nut and springs as shown in fig. A load bar extends from top of the brake and a load carrier is attached to the end of the load bar. Weight kept on this load carrier is balanced by the torque reaction in the shoes. The load arm is kept horizontal to keep the arm length constant.

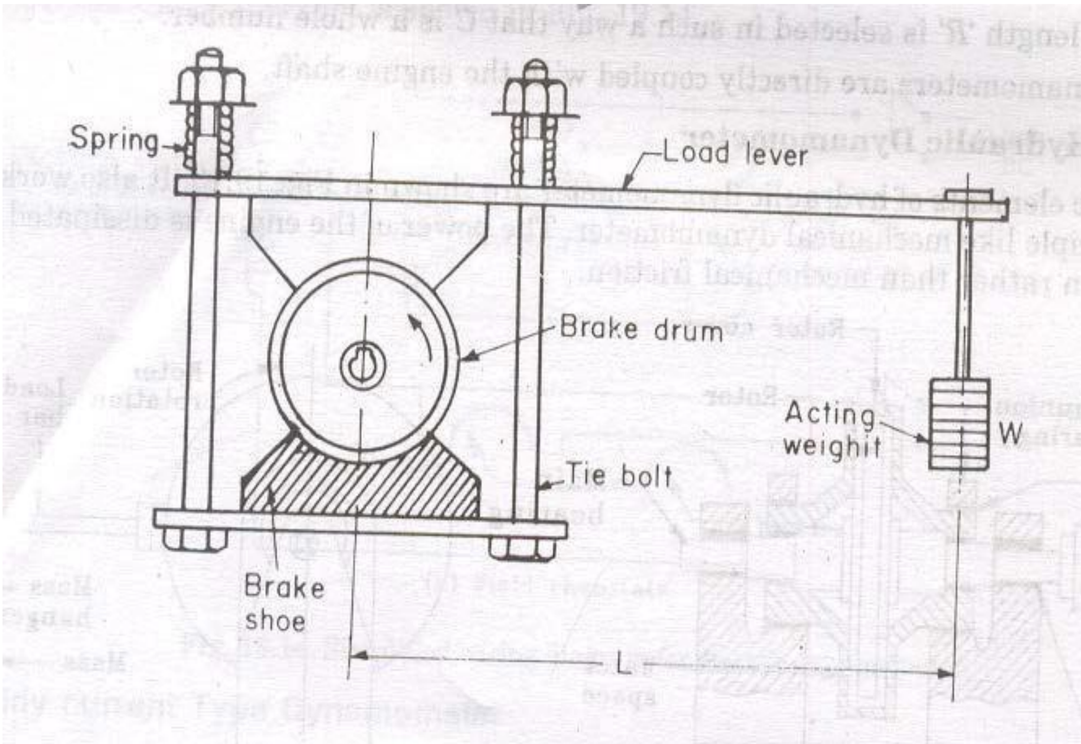


Fig. Prony-brake arrangement.

The energy supplied by engine to the brake is eventually dissipated as heat. Therefore, most of the brakes are provided with a means of supply of cooling water to the inside rim of the brake drum.

The BP of the engine is given by

B.P (brake power) = $2*\pi*N*T/60$ watts $=(2*\pi*N*T)/(60*1000)$ kW

Where T = $(W.L)$ (N-m)

Where W = Weight on load carrier, (N)

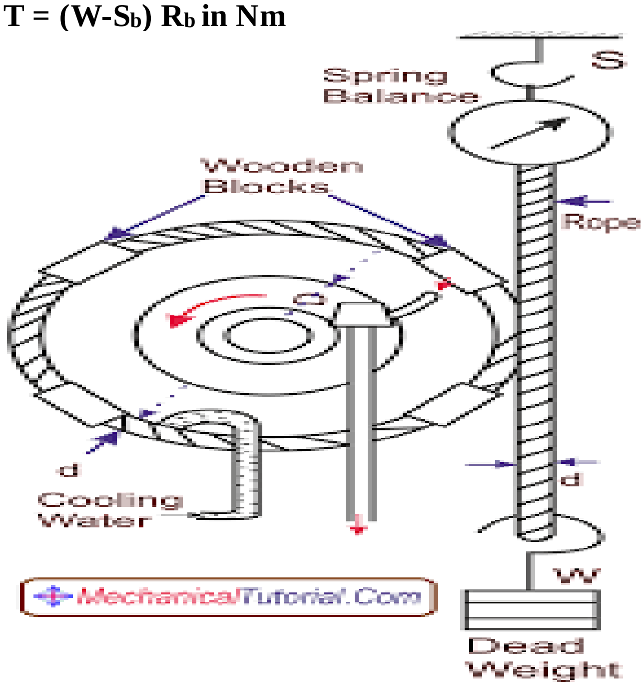
And L = Distance from the centre of shaft to the point of load-meter in meters.

The prony brake is inexpensive, simple in operation and easy to construct. It is, therefore, used extensively for testing of low speed engines. At high speeds, grabbing and chattering of the band occur and lead to difficulty in maintaining constant load. The main isadvantage of the prony brake is its constant torque at any one band pressure and therefore its inability to compensate for varying conditions.

Rope Brake dynamometer.

The rope brake as shown in Figure is another simple device for measuring torque of an engine. It consists of a number of turns of rope wound around the rotating drum attached to the output shaft. One side of the rope is connected to a spring balance and the other to a loading device. The power is absorbed in friction between the rope and the drum. The drum therefore requires cooling.

Torque= $T = (W-S_b)\ R_b$ in Nm



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S_b = Spring balance in N

W= Dead Weight in N

R_b= Effective brake radius = (Ddrum + d_{rope}) / 2 in m

($W-S_b$)= Net brake load in N

Ddrum= Diameter of drum /wheel/disc in m

d_{rope}= diameter of the rope in m

MEASUREMENT OF HEAT CARRIED AWAY BY COOLING WATER

The heat carried away by cooling water is generally measured by measuring the water flow rate through the cooling jacket and the rise in temperatures of the water during the flow through the engine.

The inlet and out let temperatures of the water are measured by the thermometers inserting in the pockets provided at inlet to and outlet from the engine. The quantity of water flowing is measured by collecting the water in a bucket for a specified period or directly with the help of flow meter in case of large engine. The heat carried away by cooling water is given by

Where	Q_w	=	$m_w Cp_w (Tw_2 - Tw_1)$ kJ/min.
	mw	=	mass of water/min.
	Tw_1	=	Inlet temperature of water, ^oC
	Tw_2	=	Out let temperature of water, ^oC
	Cp_w	=	Specific heat of water.

MEASUREMENT OF HEAT CARRIED AWAY BY EXHAUST GASES

The mass of air supplied per kg of fuel used can be calculated by using the equation if the exhaust analysis is made heat carried away by the exhaust gas per kg of fuel supplied can be calculated as

$$Q_g = m_g Cpg (Tg - Ta) \quad \text{kJ/min}$$

Where

$m_{g=} (ma + m_f)$ = mass of exhaust gases formed per kg of fuel supplied to engine Cpg = Specific heat of exhaust gases in kJ/kgK

Tg = Temperature of exhaust gases coming out from the engine ^oC.

Ta = Ambient temperature ^oC or engine room temperature.

The temperature of the exhaust gases is measured with the help of suitable thermometer or thermocouple.

Another method used for measuring the heat carried away by exhaust gases is to measure the fuel supplied per minute and also to measure the air supplied per minute with the help of air box method. The addition of fuel and air mass will be equal to the mass of exhaust gases and exhaust gas calorimeter is commonly used in the laboratory for the measurement of heat carried by exhaust gases.

HEAT BALANCE SHEET

A heat balance sheet is an account of heat supplied and heat utilized in various ways in the system. Necessary information concerning the performance of the engine is obtained from the heat balance.
The heat balance is generally done on second basis or minute basis or hour basis.
The heat supplied to the engine is only in the form of fuel-heat and that is given by

1. *Heat supplied* = $Q_s = m_f * CV$ kJ/min

Where m_f is the mass of fuel supplied per minute or per sec. and CV is the calorific value of the fuel in kJ/kg.
The various ways in which heat is used up in the system is given by

2. Heat equivalent of BP = kW = kJ/sec. = BP*60 kJ/min.

3. Heat carried away by cooling water

$$= C_{pw}\, m_w\, (Tw_2 - Tw_1) \text{ kJ/min.}$$

Where m_w is the mass of cooling water in kg/min or kg/sec circulated through the cooling jacket and ($T_{w2} - T_{w1}$) is the rise in temperature of the water passing through the cooling jacket of the engine and C_{pw} is the specific heat of water in kJ/kg-K.

4. Heat carried away by exhaust gases

$$= m_g\, C_{pg}\, (Tg - Ta) \text{ (kJ/min.)}$$

Where mg is the mass of exhaust gases in kg/min. or kg/sec and it is calculated by using mass of air and mass of fuel.

T_g = Temperature of burnt gases coming out of the engine.
T_a = Ambient or room Temperature.
C_{pg} = Sp. Heat of exhaust gases in (kJ/kg-K)

5. A part of heat is lost by convection and radiation as well as due to the leakage of gases. Part of the power developed inside the engine is also used to run the accessories as lubricating pump, cam shaft and water circulating pump. These cannot be measured precisely and so this is known as unaccounted 'losses'.

This unaccounted heat energy is calculated by the different between heat supplied Q_s and the sum of 1+2+3
Heat unaccounted =1 –(2+3+4) kJ/min.

The results of the above calculations are tabulated in a table and this table is known as "Heat Balance Sheet". It is generally practice to represent the heat distribution as percentage of heat supplied. This is also tabulated in the same heat balance sheet.

Heat input per minute	*(kJ/min)*	*%*	*Heat expenditure per minute*	*(kJ/min)*	*%*
Heat supplied by the combustion fuel	Q_s	100%	(1) Heat in BP. (2) Heat carried by jacket cooling water (3) Heat Carried by exhaust gases (4) Heat unaccounted for $= Q_s - (1 + 2 + 3)$	-- -- -- --	-- -- -- --
Total	Q_s	100%			100%

NOTE: The heat in frictional FP (IP – BP) should not be included separately in heat balance sheet because the heat of FP (frictional heat) will be dissipated in the cooling water, exhaust gases and radiation and convection. Since each of these heat quantities are separately measured and heat in FP is a hidden part of these quantities; the separate inclusion would mean that it has been included twice.

Thermal Efficiency: Thermal efficiency of an engine is defined as the ratio of the output to that of the chemical energy input in the form of fuel supply. It may be based on brake or indicated output. It is the true indication of the efficiency with which the chemical energy of fuel (input) is converted into mechanical work. Thermal efficiency also accounts for combustion efficiency, i.e., for the fact that whole of the chemical energy of the fuel is not converted into heat energy during combustion.

Brake thermal efficiency = $BP / (m_f * cv)$

where, Cv= Calorific value of fuel, kJ/kg, and
m_f = Mass of fuel supplied, kg/sec.

Indicated thermal efficiency = IP/ $(m_f * cv)$

Specific Fuel Consumption

Specific fuel consumption is defined as the amount of fuel consumed for each unit of power developed per hour. It is a clear indication of the efficiency with which the engine develops power from fuel.

Brake specific fuel consumption is defined as how much amount of fuel consumed to develop 1kW of power in one hour.

BSFC $= m_f / BP$ kg/kWhr.

Where m_f = Mass of fuel supplied, kg/hr.

similarly, ISFC = m_f / IP kg/kWhr

Mechanical Efficency: It is defined as the ratio of power output (BP) to power input(IP) of an engine. It is expressed in percentage.

Mechanical efficiency $=\eta_m= BP / IP$

Relative efficiency: It is the ratio of thermal efficiency to the corresponding air standard efficiency of an engine.

Relative efficiency$= \eta_{rel} = \eta_{th} / \eta_{air}$

If relative efficiency is based on BTE ,then relative efficiency with respect to BP .

Relative efficiency$= \eta_{rel} = \eta_{bth} / \eta_{air}$

Similarly for ITE,

Relative efficiency$= \eta_{rel} = \eta_{ith} / \eta_{air}$

Air standard efficiency corresponding to thermal efficiency of Otto or Diesel cycle.

Module-IV

RECIPROCATING COMPRESSOR

SYLLABUS: Introduction, general description and classification, volumetric efficiency, work done, need for multi staging, optimum intermediate pressure for two stage air compressor with inter-cooling, work required for multistage compressor and its efficiency.

Introduction: An air compressor is a machine which takes in atmospheric air, compresses it with the help of some mechanical energy and delivers it at high pressure. It is also called air pump. An air compressor increases the pressure of air by decreasing its specific volume using mechanical means. Thus compressed air carries an immense potential energy. The controlled expansion of compressed air provides motive force in air motors, pneumatic hammers, air drills etc.

Uses of compressed air:

The applications of compressed air are listed below:

1) It is used in gas turbines and propulsion units.
2) It is used in striking type pneumatic tools for concrete breaking, clay or rock drilling, chipping, caulking, riveting etc.
3) It is used in rotary type pneumatic tools for drilling, grinding, hammering etc.
4) Pneumatic lifts and elevators work by compressed air.
5) It is used for cleaning purposes.
6) It is used as an atomiser in paint spray and insecticides spray guns.
7) Pile drivers, extractors, concrete vibrators require compressed air.
8) Air-operated brakes are used in railways and heavy vehicles such as buses and lorries.
9) Sand blasting operation for cleaning of iron castings needs compressed air.
10) It is used for blast furnaces and air-operated chucks.
11) Compressed air is used for starting I.C.engines and also super charging them.

Classification of compressors:

01. Based on principle of operation

a. Reciprocating air compressors

i. Single acting air compressors: Single acting compressors in which suction, compression and delivery of air (or gas) take place on one side of the piston.

ii. Double acting air compressor: Double acting compressors in which suction, compression and delivery of air (or gas) take place on both sides of the piston.

b. Rotary air compressors

i. Roots blowers

i. Vane type compressors

02. Based on number of stages

a. Single stage

b. Multi stage

03. Based on Capacity

a. Low capacity compressors (up to 0.15 m^3/sec)

b. Medium capacity compressors (between 0.15 and 5 m^3/sec)

c. High capacity compressors (Greater than 5 m^3/sec)

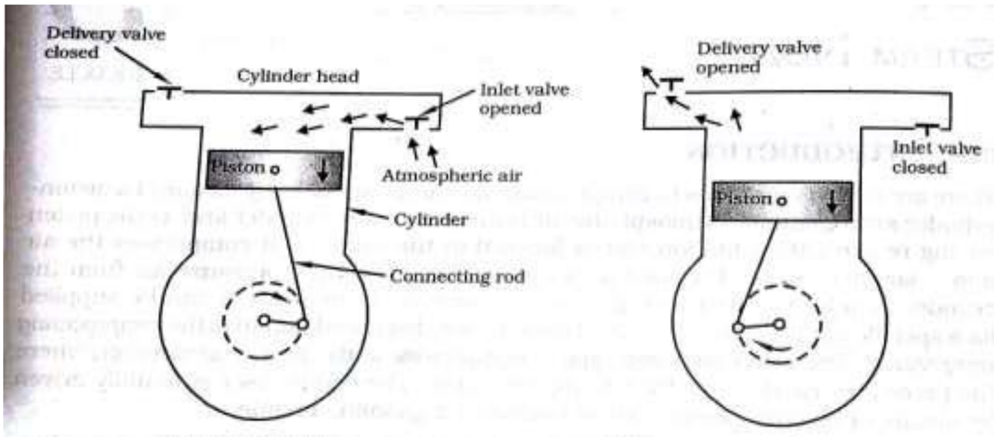
04. Based on pressure

a. Low pressure compressor

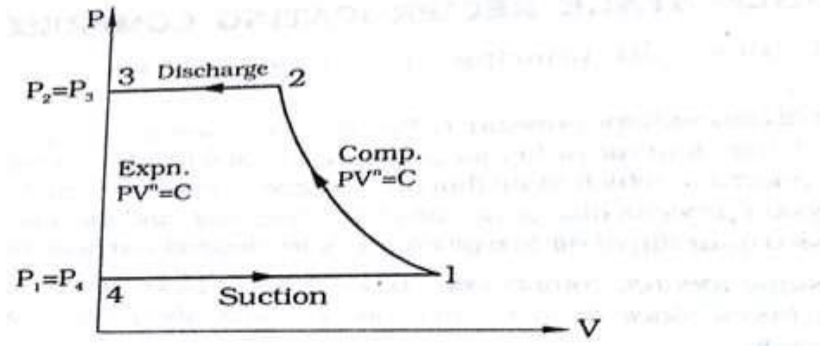
b. Medium pressure compressor

c. High pressure compressor

Working of a compressor



(a) Suction stroke (b) Compression and Delivery stroke



Theoretical P-V diagram without clearance volume

The schematic of an air compressor is as shown in figure. The working principle of a single stage air compressor with negligible clearance volume is shown in figure. It basically consists of a piston, connecting rod, crank and cylinder with valves. The working fluid may be assumed to be a perfect gas and the cycle completes in one revolution of crankshaft. The valves are spring loaded which operates due to a small pressure difference. The valves operate automatically with a light spring pressure giving a rapid closing action.

Suction Process 4-1
At the beginning, when the piston starts moving towards the bottom dead center, the pressure inside the cylinder falls below the atmospheric pressure, due to pressure difference, the inlet valve opens and atmospheric air enters the cylinder until the piston reaches the bottom dead center.

Compression process 1-2
The piston now starts moving from bottom dead center to top dead center compressing the air in the cylinder. The slight increase in the air pressure at the beginning of compression causes inlet valve to close. Since inlet valve remain closed, the upward moving piston causes the inlet valve close. Since the outlet valve remains closed, the upward moving piston compresses the air rapidly to high pressures.

Delivery Process 2-3
When the air pressure in the cylinder is slightly above the receiver tank pressure, the deliver outlet valve opens. The high pressure air in the cylinder is delivered during the upward motion of the piston and process is shown by the line 2-3 on P-V Diagram. Thus the cycle is completed.

Process 3-4: No air in the cylinder and return of piston for suction stroke.

Disadvantages

1. Handling of high pressure air results in leakage through the piston.
2. Cooling of the gas *is* not effective.
3. Requires a stronger cylinder to withstand high delivery pressure.

Reciprocating Compressor Terminology

1. Single acting compressor is a compressor in which suction, compression and delivery of a gas take place only on one side of the piston during a cycle of one revolution of the crank shaft.
2. Double acting compressor is a compressor in which suction, compression and delivery of a gas take place on both sides of the piston and two cycles take place during one revolution of the crank shaft.
3. Single stage compressor is a compressor in which the compression of gas to final delivery pressure is carried out in one cylinder only.
4. Multistage compressor is a compressor in which the compression of gas to final delivery pressure is carried out in more than one cylinder in series.
5. Pressure ratio is defined as the absolute discharge pressure to absolute suction pressure.
6. Free air is the air that exists under atmospheric condition. If the actual volume of air delivered at given pressure expressed in terms suction (intake) pressure and temperature, then the air delivered in m^3/sec, is known as free air delivered.
7. Compressor displacement volume is the volume created when the piston travels a stroke. It is given by d= bore of the cylinder, L= stroke of the piston

   $Vs = AL \text{ in } m^3 = \frac{\prod}{4} \times d^2 \times L \times \frac{N}{60}$ in m^3/sec for single acting

   $Vs = AL * 2 = \frac{\prod}{4} \times d^2 \times L \times \frac{N}{60} * 2$ for double acting
8. Capacity of compressor is the actual quantity of air delivered per unit at atmospheric conditions.
9. Clearance volume is the volume remains in the cylinder after the piston has reached its dead end during compression stroke.
10. Actual (effective) swept volume is the actual amount of free air delivered by the cylinder during one cycle in one second.
11. Clearance ratio is the ratio of clearance volume to swept volume.

$$C \text{ or } K = \frac{\text{Clearance volume}}{\text{Swept volume}}$$

12. Piston speed is the linear speed of the piston measured in m/min. it is expressed as

$$V_{piston} = 2LN$$

Compression processes:

The air may be compressed by the following processes.

(a) Isentropic or adiabatic compression,

(b) Polytropic compression and

(c) Isothermal compression

(a)* Isentropic (or) adiabatic compression*:

In internal combustion engines, the air (or air fuel mixture) is compressed isentropically.
By isentropic compression, maximum available energy in the gas is obtained.

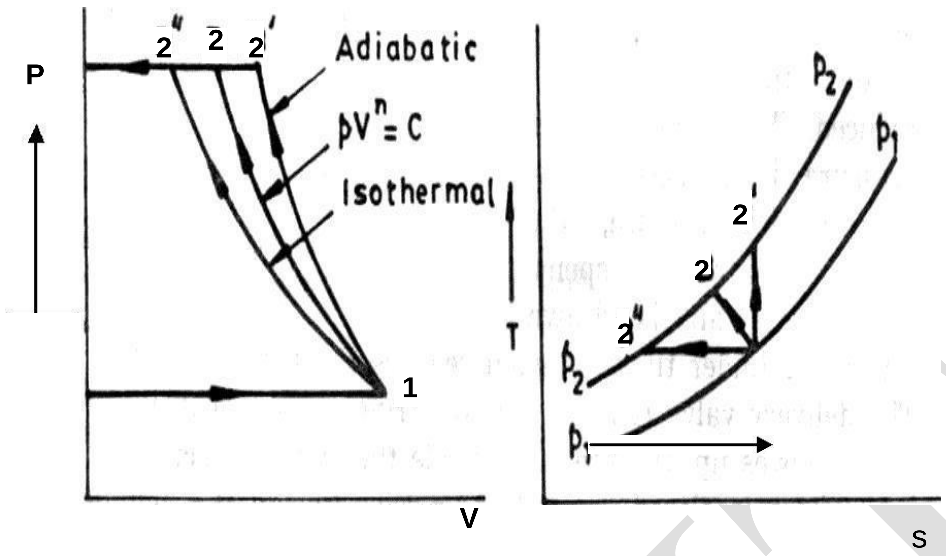


Fig: Compression processes
1-2": Isothermal; 1-2: Polytropic; 1-2': Isentropic

The compression follows the law PV^n = Constant. This type of compression may be used in Bell-Coleman cycle of refrigeration.

(b)Isothermal compression:

When compressed air (or gas) is stored in a tank, it loses its heat to the surroundings. It attains the temperature of surroundings after some time. Hence, the overall effect of this compression process is to increase the pressure of the gas keeping the temperature constant. Thus isothermal compression is suitable if the compressed air (or gas) is to be stored.

Power required for driving the compressor:

The following assumptions are made in deriving the power required to drive the compressor.

1. There is no pressure drop through suction and delivery valves.
2. Complete compression process takes place in one cylinder.
3. There is no clearance volume in the compressor cylinder.
4. Pressure in the suction line remains constant. Similarly, pressure in the delivery line remains constant.
5. The working fluid behaves as a perfect gas.
6. There is no frictional losses.

The cycle can be analysed for the three different case of compression. Work required can be obtained from the p - V diagram.

Let, P_1 = Pressure of the air (kN/m^2), before compression.
V_1 = Volume of the air (m^3), before compression
T_1 =Temperature of the air *(K),* before compression
P_2, V_2 and T_2 be the corresponding values after compression.
m = Mass of air induced or delivered by the cycle (kg).
N = Speed in RPM.

Polytropic Compression (Work done of a compressor without clearance)

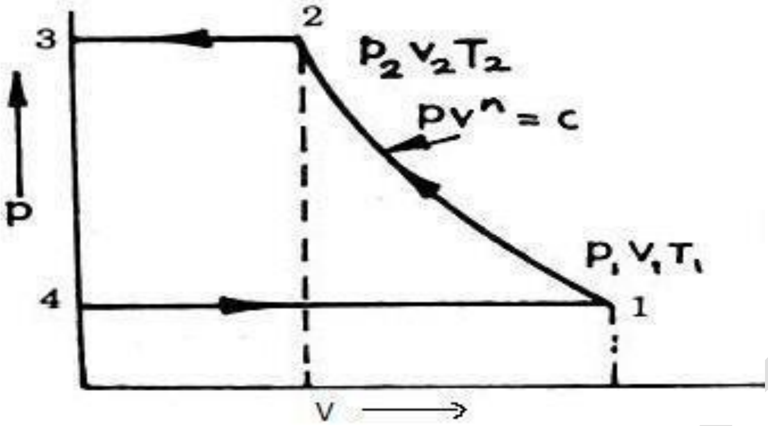


Fig: Polytropic compression (Compression follows pV = Constant)

Let n= Index of polytropic compression

Net work done on air/cycle is given by

W = Area 1-2-3-4-1

= Work done during compression (1-2) + Work done during air delivery (2-3) - Work done during suction (4-1).

$$W = \frac{p_2v_2 - p_1v_1}{n-1} + p_2v_2 - p_1v_1$$

$$W = \frac{p_2v_2 - p_1 + (n-1)p_2v_2 - (n-1)p_1v_1}{n-1}$$

$$= \frac{np_2v_2 - np_1v_1}{n-1} = \left(\frac{n}{n-1}\right) p_2v_2 - p_1v_1$$

We know that, $p_1V_1 = m\,RT_1$ & $p_2V_2 = m\,RT_2$

Therefore, $\mathbf{W = \frac{n}{n-1}\, m\, R\, (T_2 - T_1)}$

$$\mathbf{W = \frac{n}{n-1}\, m\, R\, T_1 \left[\frac{T_2}{T_1} - 1\right]}$$

For polytropic process, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

Therefore, $W = \frac{n}{n-1}\, m\, R\, T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$ kJ/cycle

$$W = \frac{n}{n-1}\, p_1\, V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

Indicated power (or) Power required, P = W x N in kW for single acting

= W x 2N in kW for double acting reciprocating compressor.

Isentropic compression

Compression follows, pV^{γ}= Constant

Let γ = Index of isentropic compression

Net work done on air/cycle is given by

W = Area 1-2-3-4-1

= Work done during compression (1-2) + Work done during air delivery (2-3) - Wc done during suction (4-1).

$$W = \frac{p_2v_2 - p_1v_1}{\gamma - 1} + p_2v_2 - p_1v_1$$

$$W = \frac{p_2v_2 - p_1 + (\gamma-1)p_2v_2 - (\gamma-1)p_1v_1}{\gamma-1}$$

$$= \frac{\gamma p_2v_2 - \gamma p_1v_1}{\gamma-1} = \left(\frac{\gamma}{\gamma-1}\right) p_2v_2 - p_1v_1$$

We know that, $p_1V_1 = m\,RT_1$ & $p_2V_2 = m\,RT_2$

$$\mathbf{W = \frac{\gamma}{\gamma-1}\, m\, R\, (T_2 - T_1)}$$

$$\mathbf{W = \frac{\gamma}{\gamma-1}\, m\, R\, T_1 \left[\frac{T_2}{T_1} - 1\right]}$$

For isentropic process, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

Therefore, $W = \frac{\gamma}{\gamma-1}\, m\, R\, T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$ kJ/cycle

$$W = \frac{\gamma}{\gamma-1}\, p_1\, V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \text{ kJ/cycle}$$

Isothermal Compression

Compression follows, pV= Constant

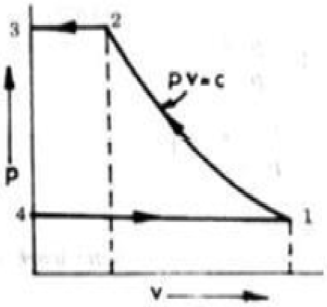


Fig: 4.5 Isothermal Compression

Isothermal Work input, W = Area 1-2-3-4-1 = area under 1-2 + area under 2-3 - area under 4-1

$$W = p_1V_1\, ln\left(\frac{V_1}{V_2}\right) + p_2V_2 - p_1V_1$$

But $p_1V_1 = p_2V_2$

$$W = p_1V_1 \; ln\left(\frac{V_1}{V_2}\right) \qquad and \; \frac{V_1}{V_2} = \frac{p_2}{p_1}$$

$$\text{Therefore, } W = p_2V_2 \; ln\left(\frac{p_2}{p_1}\right) \text{ kJ/cycle}$$

Isothermal efficiency: Isothermal efficiency is defined as the ratio of isothermal work input to the actual work input. This is used for comparing the compressors.

$$\text{Isothermal efficiency, } \eta_{iso} = \frac{Isothermal\ work\ input}{Actual\ work\ output}$$

Adiabatic efficiency: Adiabatic efficiency is defined as the ratio of adiabatic work input to the actual work input. This is used for comparing the compressors.

$$\text{Adiabatic efficiency, } \eta_{adia} = \frac{Adiabatic\ work\ input}{Actual\ work\ output}$$

Mechanical efficiency:
The compressor is driven by a prime mover. The power input to the compressor is the shaft power (brake power) of the prime mover. This is also known as brake power of the compressor.

Mechanical efficiency is defined as the ratio of indicated power of the compressor to the power input to the compressor.

$$\eta_m = \frac{\text{Indicated power of compressor}}{\text{Power input}}$$

Indicated Power, IP $= \frac{p_m l\, aNk}{60}$,
where, p_m = mean effective pressure, kN/m²
l = length of stroke of piston, m
a = area of cross section of cylinder, m²
N= crank speed in rpm, and
K =number of cylinders

Clearance and clearance volume:
When the piston reaches top dead centre (TDC) in the cylinder, there is a dead space between piston top and the cylinder head. This space is known as clearance space and the volume occupied by this space is known as clearance volume, V_C.

The clearance volume is expressed as percentage of piston displacement. Its value ranges from 5% - 10% of swept volume or stroke volume (V_S).The p - V diagram for a single stage compressor, considering clearance volume is shown in fig. At the end of delivery of high pressure air (at point 3), a small amount of high pressure air at p_2 remains in the clearance space. This high pressure air which remains at the clearance space when the piston is at TDC is known as remnant air. It is expanded polytropically till atmospheric pressure (p_4=p_1) is reached. The inlet valve is opened and the fresh air is sucked into the cylinder. The suction of air takes place for the rest of stroke (upto point 1). The volume of air sucked is known as effective suction volume (V_1 - V_4). At point 1, the air is compressed polytropically till the delivery pressure (p_2) is reached. Then the delivery valve is opened and high pressure air is discharged into the receiver. The delivery of air continues till the piston reaches its top dead centre, then the cycle is repeated.

Effect of clearance volume:

The following are the effects of clearance space.

1. Suction volume (volume of air sucked) is reduced.
2. Mass of air is reduced.
3. If clearance volume increases, heavy compression is required.
4. Heavy compression increases mechanical losses.

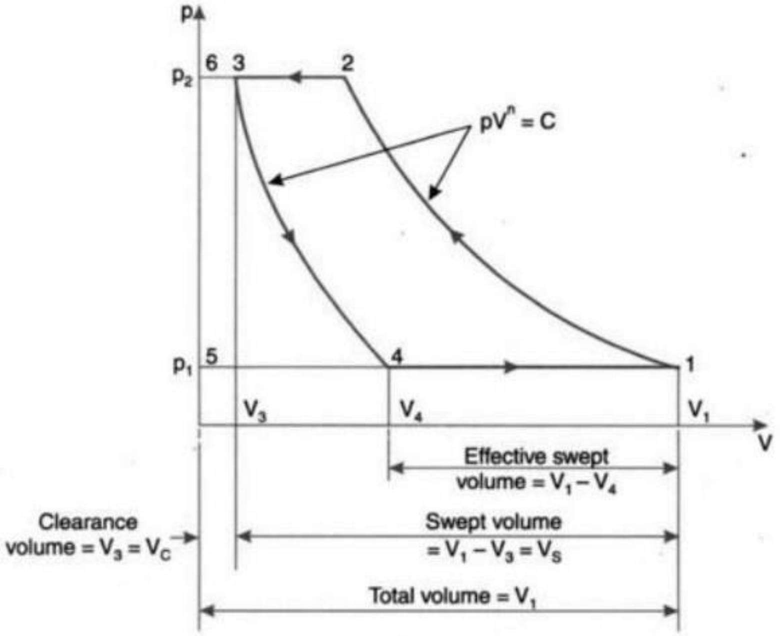


Fig: p-V diagram with clearance volume

Work input considering clearance volume:

Assuming the expansion (3-4) and compression (1-2) follow the law $pV^n = C$,
Work input per cycle is given by,
W = Area (1-2-3-6-5-4-1)- Area (3-6-5-4 -3)
W = Workdone during compression - Work done during expansion

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1} p_4 V_4 \left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right]$$

But, $p_3 = p_2$ and $p_4 = p_1$
therefore

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1} p_1 V_4 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

V_1-V_4 is called as effective suction volume.

Volumetric efficiency:

The clearance volume in a compressor reduces the intake capacity of the cylinder. This leads to a term called volumetric efficiency.

The volumetric efficiency is denned as the volume of free air sucked into the compressor per cycle to the stroke volume of the cylinder,the volume measured at the intake pressure and temperature or at standard atmospheric conditions,(p_s =101.325 kN/m^2 and T_s = 288K)

$$\text{Volumetric efficiency, } \eta_{vol} = \frac{\text{Volume of free air taken in per cycle}}{\text{Stroke volume of the cylinder}}$$

$$= \frac{\text{Effective suction volume}}{\text{Swept volume}} = \frac{(V1 - V4)}{(V1 - V3)} = \frac{V_1 - V_4}{V_s}$$

Clearance ratio: Clearance ratio is defined as, the ratio of clearance volume to swept volume. It is denoted by the letter C.

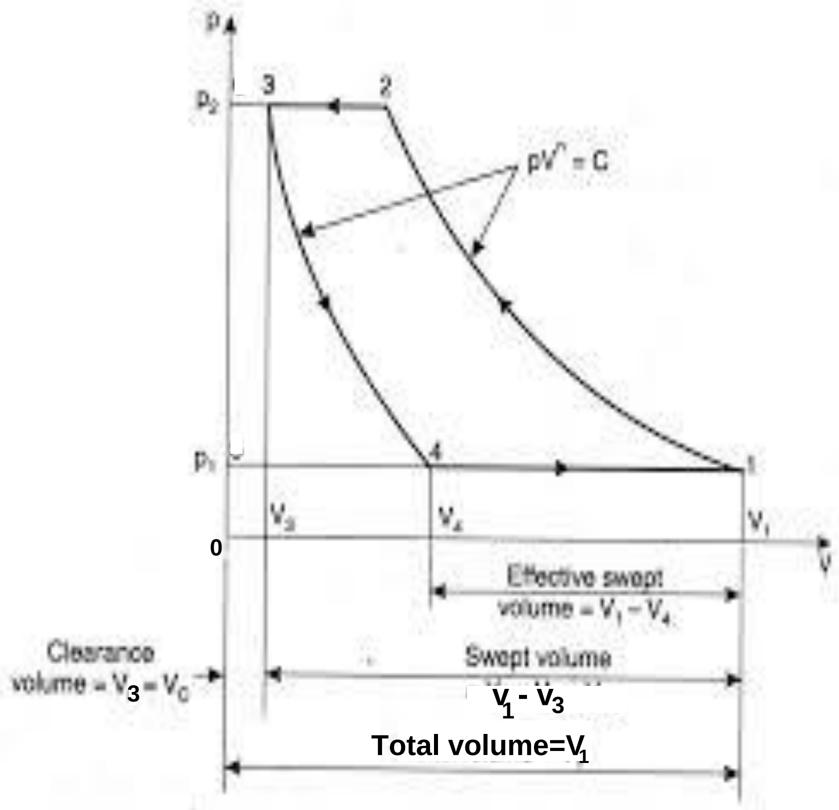
$$\text{Clearance ratio,C} = \frac{Clearance\ volume}{Swept\ volume} = \frac{V_c}{V_s} = \frac{V_c}{V_{1-V_3}}$$

$$\text{Pressure ratio, } R_p = \frac{Delivery\ pressure}{Suction\ pressure} = \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

Expression for Volumetric efficiency

Let the compression and expansion follows the law, pV^n =Constant.

$$\text{Clearance ratio, C} = \frac{Clearance\ volume}{Swept\ volume} = \frac{V_c}{V_s} = \frac{V_3}{V_{1-V_3}}$$



$V_1 - V_3 = \frac{V_3}{C}$ -------------(1)

$V_1 = \frac{V_3}{C} + V_3$

$V_1 = V_3 \left(\frac{1}{C} + 1\right)$ --------------------- (2)

We know that, Pressure ratio, $R_p = \frac{Delivery\ pressure}{Suction\ pressure} = \frac{p_2}{p_1} = \frac{p_3}{p_4}$

By polytropic expansion process 3-4:

$$\frac{p_3}{p_4} = \left(\frac{V_4}{V_3}\right)^n$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n} = (R_p)^{\frac{1}{n}}$$

Therefore, $V_4 = V_3 (R_p)^{\frac{1}{n}}$ ----------------- (3)

Volumetric efficiency, $\eta_{vol} = \frac{\text{Effective suction volume}}{\text{Swept volume}} = \frac{(V_1 - V_4)}{(V_1 - V_3)}$ ------------ (4)

Using equations 1,2 and 3 in 4,

$$\eta_{vol} = \frac{V_3\left(\frac{1}{C}+1\right) - V_3[R_p]^{1/n}}{\frac{V_3}{C}} = \frac{V_3\left\{\left(\frac{1}{C}+1\right) - [R_p]^{1/n}\right\}}{V_3\left(\frac{1}{C}\right)} = \frac{\left\{\left(\frac{1}{C}+1\right) - [R_p]^{1/n}\right\}}{\left(\frac{1}{C}\right)} = C\left[\left(\frac{1}{C}+1\right) - [R_p]^{1/n}\right]$$

$$\eta_{vol} = 1 + C - C[R_p]^{1/n} = 1 + C - C\left[\frac{p_2}{p_1}\right]^{1/n}$$

Limitations of single stage reciprocating compressor

When a single stage compressor is required to deliver a high pressure compressed air it suffers from following limitations

01. Cylinder wall thickness increases making the compressor heavy.
02. Requires heavy moving components to compress air to high pressure
03. Use of heavy working components increases balancing problem.
04. Temperature of air at deliver increases. High temperatures may heat up the cylinder head and burn the lubricating oil.
05. Work input increases with increase in pressure ratio.
06. Volumetric efficiency decreases with increase in pressure ratio which effects the air handling capacity of compressor adversely

Need for intercooling in multi stage compression

In a multi stage compressor the air is compressed in succeeding cylinders, and as a result, the temperature of air increases to higher levels after every stage. Thus temperature of air is needed to be reduced and this is achieved with an intercooler.

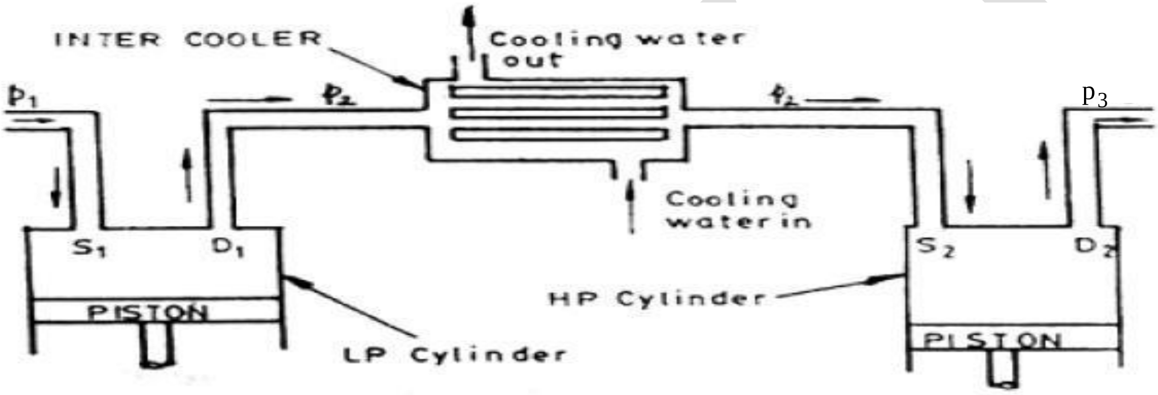
An intercooler is a heat exchanger which cools the compressed air to a safe value, so that it can be compressed again and again depending on the requirement. Usually intercooler is installed between the cylinders in each stage to cool the air before it is ingested into the next cylinder for further compression.

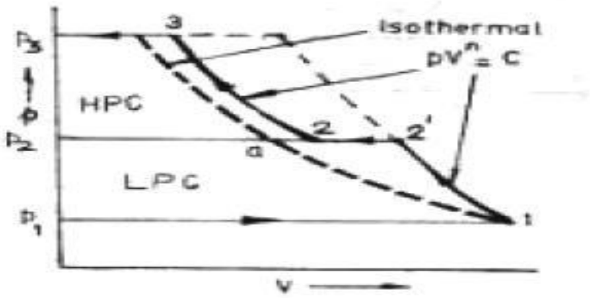
Multi-stage air compressor:

In a multi stage air compressor, compression of air takes place in more than one cylinder. Multi stage air compressor is used in places where high pressure air is required. Fig. shows the general arrangement of a two-stage air compressor. It consists of a low pressure (L.P) cylinder, an intercooler and a high pressure (H.P) cylinder. Both the pistons (in L.P and H.P cylinders) are driven by a single prime mover through a common shaft.

Atmospheric air at pressure p_1 taken into the low pressure cylinder is compressed to a high pressure (p_2). This pressure is intermediate between intake pressure (p_1) and delivery pressure p_3). Hence this is known as intermediate pressure.

The air from low pressure cylinder is then passed into an intercooler. In the intercooler, the air is cooled at constant pressure by circulating cold water. The cooled air from the intercooler is then taken into the high pressure cylinder. In the high pressure cylinder, air is further compressed to the final delivery pressure (p_3) and supplied to the air receiver tank.





Advantages:

1. Saving in work input: The air is cooled in an intercooler before entering the high pressure cylinder. Hence less power is required to drive a multistage compressor as compared to a single stage compressor for delivering same quantity of air at the same delivery pressure.

2. Better balancing: When the air is sucked in one cylinder, there is compression in the other cylinder. This provides more uniform torque. Hence size of the flywheel is reduced.

3. No leakage and better lubrication: The pressure and temperature ranges are kept within desirable limits. This results in a) Minimum air leakage through the piston of the cylinder and b) effective lubrication due to lower temperature.

4. More volumetric efficiency: For small pressure range, effect of expansion of the remnant air (high pressure air in the clearance space) is less. Thus by increasing number of stages, volumetric efficiency is improved.

5. High delivery pressure: The delivery pressure of air is high with reasonable volumetric efficiency.

6. Simple construction of LP cylinder: The maximum pressure in the low pressure cylinder is less. Hence, low pressure cylinder can be made lighter in construction.

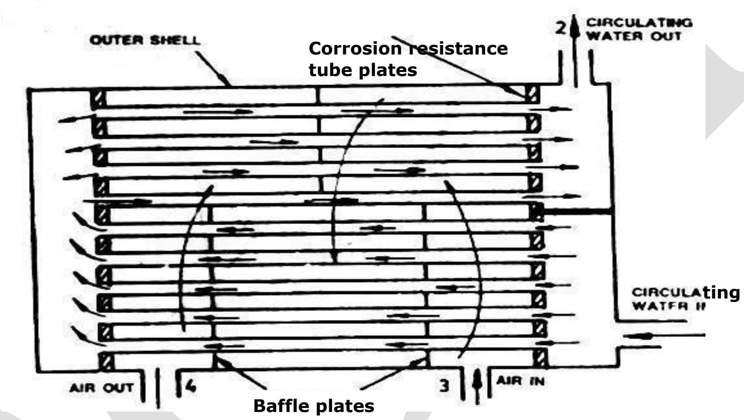
7. Cheaper materials: Lower operating temperature permits the use of cheaper materials for construction.

Disadvantages:

1. More than one cylinder is required.
2 An intercooler is required. This increases initial cost. Also space required is more.
3. Continuous flow of cooling water is required.
4. Complicated in construction.

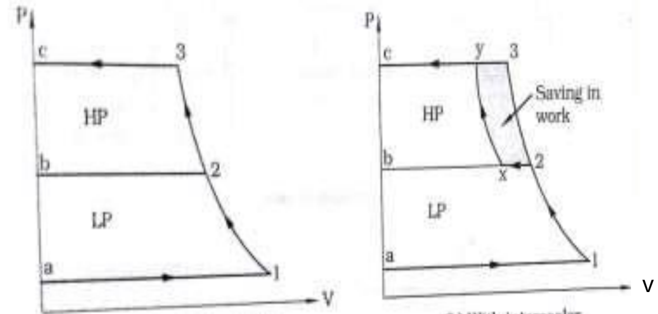
Intercoolers:

An intercooler is a simple heat exchanger. It exchanges the heat of compressed air from the LP compressor to the circulating water before the air enters the HP compressor. It consists of a number of special metal tubes connected to corrosion resistant plates at both ends. The entire nest of tubes is covered by an outer shell.



Working: Cold water enters the bottom of the intercooler through water inlet (1) and flows into the bottom tubes. Then they pass through the top tubes and leaves through the water outlet (2) at the top. Air from LP compressor enters through the air inlet (3) of the intercooler and passes over the tubes. While passing over the tubes, the air is cooled (by the cold water circulated through the tubes). This cold air leaves the intercooler through the air outlet (4). Baffle plates are provided in the intercooler to change the direction of air. This provides a better heat transfer from air to the circulating water.

Work savings with inter cooler



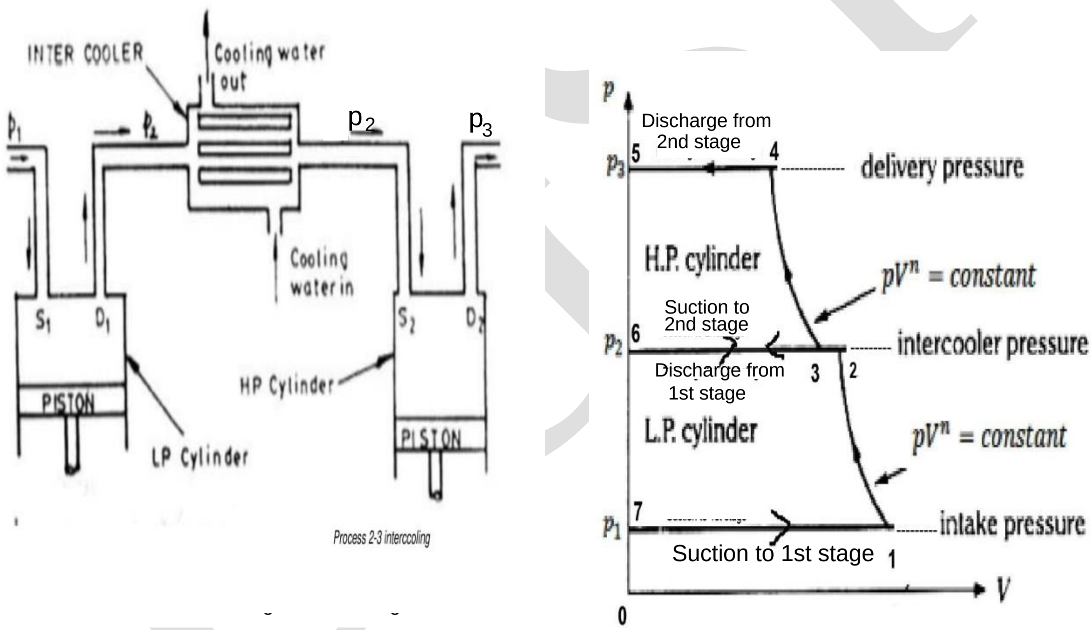
(a) Without intercooler (b) With intercooler

Figure shows the P-V diagram of two stage compressor with and without intercooler. The net work done in compressing the air from 1-3 is given by sum of work done in each stage.
The net work done by compressor without intercooler in compressing the air from 1-3 is given by sum of work done in each stages i.e, net work done is the sum of work done in low pressure cylinder and high pressure cylinder. The work done in low pressure cylinder is given by the area (1-2-b-a-1) and work done by high pressure cylinder is given by area (2-3-c-b-2).
Figure b shows a similar compressor with inter cooler. The compressed air leaving the low pressure cylinder at state 2 enters inter cooler, gets cooled till condition X is reached and then enters high pressure cylinder. The work done in low pressure cylinder remains same and is given by the area (1-2-b-a-1). But work done by high pressure cylinder is given by X-Y-c-b-X which is less than 2-3-c-b-2.

Work input required in multistage compressor:

The following assumptions are made for calculating the work input in multistage compression.

1. Pressure during suction and delivery remains constant in each stage
2. Intercooling takes place at constant pressure in each stage
3. The compression process is same for each stage.
4. The mass of air handled by LP cylinder and HP cylinder is same.
5. There is no clearance volume in each cylinder.
6. There is no pressure drop between the two stages.



Work required to drive the multi-stage compressor can be calculated from the area of the p - V diagram.

Let, p_1, V_1 and T_1 be the condition of air entering the LP cylinder. P_2, V_2 and T_2 be the condition of air entering the HP cylinder. P_3 be the final delivery pressure of air.

Then,
Total work input = Work input for LP compressor + Work input for HP compressor.

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

$$W = \frac{n}{n-1} m R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} m R T_2 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

If intercooling is perfect, $T_2 = T_1$, therefore,

$$W = \frac{n}{n-1} m R T_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} m R T_1\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right] \text{ kJ/cycle}$$

$$W = \frac{n}{n-1} m R T_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] \text{ kJ/cycle}$$

4.16 Condition for maximum efficiency (or)
Condition for minimum work input (or)
To prove that for minimum work input the intermediate pressure of a two-stage compressor with perfect intercooling is the geometric mean of the intake pressure and delivery pressure (or)
To prove $p_2 = \sqrt{p_1 p_3}$

Work input for a two-stage air compressor with perfect intercooling is given by,

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] \text{ kJ/cycle}$$

If the initial pressure (**p_1**) and final pressure (**p_3**) are fixed, the value of intermediate pressure (**p_2**) can be determined by differentiating the above equation of work input in terms of **p_2** and equating it to zero.

Let, $\frac{n}{n-1} p_1 V_1 = k\ (constant)$ and $\frac{n-1}{n} = a$

then,

$$W = k\left[\left(\frac{p_2}{p_1}\right)^a + \left(\frac{p_3}{p_2}\right)^a - 2\right]$$

or

$$W = k(p_2^a\, p_1^{-a} + p_3^a p_2^{-a} - 2) \quad \text{--------------------- (1)}$$

Differentiating the above equation (1) with respect to p_2 and equating it to zero,

$$\frac{dW}{dp_2} = k\, a\, p_2^{a-1}\, p_1^{-a} + k\,(-a) p_3^a\, p_2^{-a-1} = 0$$

$$k\, a\, \frac{p_2^a}{p_2 p_1^a} - k\, a\, p_3^a\, \frac{1}{p_2^a\, p_2} = 0$$

or

$$\frac{k\ a\ p_2^a}{p_2\, p_1^a} = \frac{k\ a\ p_3^a}{p_2\, p_2^a}$$

$$\left(\frac{p_2}{p_1}\right)^a = \left(\frac{p_3}{p_2}\right)^a$$

$=> \quad p_2^2 = p_1 p_3$

or

intermediate pressure, $\mathbf{p_2} = \sqrt{\mathbf{p_1 p_3}}$

Thus for maximum efficiency the intermediate pressure is the geometric mean of the initial and final pressures.

4.17 Minimum work input for multistage compression with perfect intercooling:

Work input for a two-stage compressor with perfect intercooling is given by

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right] \text{ ------------------------ (1)}$$

Work input will be minimum if $\frac{p_2}{p_1} = \frac{p_3}{p_2}$ ------------------------------ (2)

$$p_2^2 = p_1 \, p_3$$

Dividing both sides by p_1^2 ,

$$\left(\frac{p_2}{p_1}\right)^2 = \frac{p_3}{p_1} \qquad \Rightarrow \frac{p_2}{p_1} = \left(\frac{p_3}{p_1}\right)^{1/2} \text{ -------------------------------- (3)}$$

From (2), $\frac{p_3}{p_2} = \frac{p_2}{p_1} = \left(\frac{p_3}{p_1}\right)^{1/2}$ --------------------------------------- (4)

Substituting the equation (4) in equation (1), work input for a two stage compressor,

$$W_{min} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_1}\right)^{1/2\left[\frac{n-1}{n}\right]} + \left(\frac{p_3}{p_1}\right)^{1/2\left[\frac{n-1}{n}\right]} - 2 \right]$$

$$= \frac{n}{n-1} p_1 V_1 \left[2\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 2 \right]$$

$$W_{min} = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right]$$

or

$$W_{min} = \frac{2n}{n-1} mRT_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right]$$

For a three stage compressor,

$$W_{min} = \frac{3n}{n-1} p_1 V_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{3n}} - 1 \right]$$

or

$$W_{min} = \frac{3n}{n-1} mRT_1 \left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{3n}} - 1 \right]$$

Generally, the minimum work input for a multistage reciprocating air compressor with x number of stages is given by,

$$W_{min} = \frac{xn}{n-1} p_1 V_1 \left[\left(\frac{p_{x+1}}{p_1}\right)^{\frac{n-1}{xn}} - 1 \right]$$

Minimum work input required for a two stage reciprocating air compressor with perfect intercooling is given by,

$$W_{min} = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_3}{p_1} \right)^{\frac{n-1}{2n}} - 1 \right] kJ$$

But,from equation (4), $\left(\frac{p_3}{p_1}\right)^{1/2} = \frac{p_2}{p_1}$

Therefore,

$$\boldsymbol{W_{min} = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] kJ}$$

So, for maximum efficiency ie., for minimum work input, the work required for each stage is same.

For maximum efficiency, the following conditions must be satisfied:

1. The air is cooled to the initial temperature between the stages (Perfect cooling between stages).
2. In each stage, the pressure ratio is same. $\left(\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \cdots \right)$
3. The work input for each stage is same.

For an ideal compressor with X number of stages Pressure ratio is given by

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} \text{------------} \frac{P_{x+1}}{P_x} = Z, \text{ say}$$

$P_2 = Z\, P_1$

$P_3 = Z\, P_2 = Z\,(ZP_1) = Z^2\, P_1$

Similarly $P_4 = Z^3\, P_1$

$P_5 = Z^4\, P_1$

$P_x = Z^{x-1}\, P_1$

$\frac{P_{x+1}}{P_x} = Z$

$P_{x+1} = Z\, P_1 = Z(Z^{x-1}P_1) = Z^x P_1$

$\frac{P_{x+1}}{P_1} = Z^x$

$Z = \left(\frac{P_{x+1}}{P_1} \right)^{1/x}$

RECIPROCATING COMPRESSOR
NUMERICALS

1. A reciprocating air compressor has cylinder with 24 cm bore and 36 cm stroke. Compressor admits air at 1 bar, 17°C and compresses it up to 6 bar. Compressor runs at 120 rpm. Considering compressor to be single acting and single stage determine mean effective pressure and the power required to run compressor when it compresses following the isothermal process and polytropic process with index of 1.3. Also find isothermal efficiency when compression is of polytropic and adiabatic type.

2. A single stage single acting reciprocating air compressor has air entering at 1 bar, 20°C and compression occurs following polytropic process with index 1.2 upto the delivery pressure of 12 bar. The compressor runs at the speed of 240 rpm and has L/D ratio of 1.8. The compressor has mechanical efficiency of 0.88. Determine the isothermal efficiency and cylinder dimensions. Also find out the rating of drive required to run the compressor which admits 1 m^3 of air per minute.

3. A reciprocating air compressor has four stage compression with 2 m3/min of air being delivered at 150 bar when initial pressure and temperature are 1 bar, 27°C. Compression occur polytropically following polytropic index of 1.25 in four stages with perfect intercooling between stages. For the optimum intercooling conditions determine the intermediate pressures and the work required for driving compressor.

4. A two stage reciprocating air compressor has air being admitted at 1 bar, 27°C and delivered at 30 bar, 150°C with inter stage pressure of 6 bar and intercooling up to 35°C. Compressor delivers at the rate of 2 kg/s. Clearance volumes of LP and HP cylinders are 5% and 7% of stroke volume respectively. The index of compression and expansion are same throughout. Determine the swept volume of both cylinders in m3/min, amount of cooling required in intercooler and total power required. Also estimate the amount of cooling required in each cylinder.

5. A single stage double-acting air compressor is required to deliver 14 m^3 of air per minute at 1.013 bar and 15°C. The delivery pressure is 7 bar and the speed 300 rpm. Take the clearance volume as 5% of the swept volume with the compression and expansion index n = 1.3, calculate (i) Swept volume of cylinder (ii) Indicated power.

6. A multistage compressor is to be designed to elevate the pressure from 1 bar to 120 bar, such that the stage pressure ratio will not exceed 4. Determine (i) Number of stages (ii) Minimum power required (iii) Intermediate pressures (iv) Exact pressure ratio. It is required to compress 15 m3/min of free air. Take n = 1.2.

7. A single stage single acting air compressor has cylinder bore 15cm and piston stroke of 25cm. The crank speed is 600rpm. The air taken from the atmospheric is at 1bar and 27°C and delivered at 11bar. Assuming both compression and expansion processes are according to $PV^{1.25}$=C and clearance is 5%. Determine i) Power required to drive the compressor, assuming mechanical efficiency is 80%, ii) What will be change in power required to drive the compressor if clearance is 10% with the other conditions are same.

8. An air compressor takes in air at 1bar and 20°C and compresses the same according to the law $PV^{1.2}$=C. It is delivered to a receiver at a constant pressure of 10bar. Determine i) Temperature at the end of compression ii) Work done and heat transferred during compression per kg of air.

9. A two stage air compressor with perfect inter cooling takes in air at 1bar and 27°C. The law of compression in both the stages is $PV^{1.3}$=C. The compressed air is delivered at 9bar. Calculate for unit mass flow rate of air the minimum work done and the heat rejected to the intercooler. Compare the values if the compression is carried out in single stage compressor.

10. A reciprocating compressor of single stage and double acting type is running at 200 rpm with mechanical efficiency of 85%. Air flows into compressor at the rate of 5 m^3/min measured at atmospheric condition of 1.02 bar, 27°C. Compressor has compressed air leaving at 8 bar with compression following polytropic process with index of 1.3. Compressor has clearance volume of 5% of stroke volume. During suction of air from atmosphere into compressor its temperature rises by 10°C. There occurs pressure loss of 0.03 bar during suction and pressure loss of 0.05 bar during discharge passage through valves. Determine the dimensions of cylinder, volumetric efficiency and power input required to drive the compressor if stroke to bore ratio is 1.5.

MODULE-V

Refrigeration and Air condition

Refrigeration

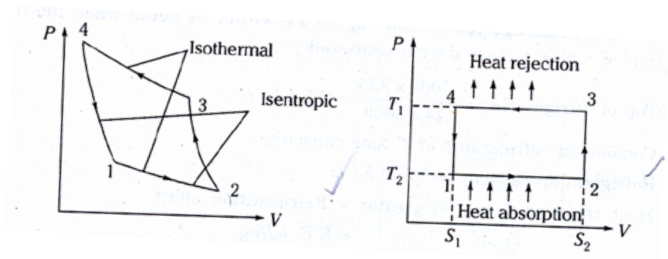
<u>Definition</u>

Refrigeration is the process of providing and maintaining temperature of the system below that of the surrounding atmosphere.

<u>Carnot Cycle</u>

The reversed carnot cycle can be considered in refrigeration system.

$$C.O.P = \frac{T_2}{T_2 - T_1} \quad \text{where } T_2 < T_1$$



Unit of Refrigeration

The common unit used in the field of refrigeration is known as Ton of refrigeration.

A ton of refrigeration is defined as the quantity of heat required to be removed to produce one ton (1000kg) of ice within 24 hours when the initial condition of water is 0°C

$$Ton \text{ of refrigeration} = \frac{1000x335}{24x3600} = 3.5 \text{ kJ/s}$$

Consider a refrigerator of T tons capacity,
Refrigeration capacity = 3.5 kJ/s
Heat removed from
refrigerator = Refrigeration effect =R.E. kJ/s
Power of the compressor =work/kg of refrigerant x mass flow rate

Air Refrigeration system working on
Bell-coleman cycle

In air refrigeration system, air is used as the refrigerant which always remains in the gaseous phase. The heat removed consists only of sensible heat and as a result, the coefficient of performance (C.O.P) is low.
The various processes are:

Process 1-2:

The air leaving the evaporator enters a compressor. Where it is compressed isentropically to higher pressure and temperature.

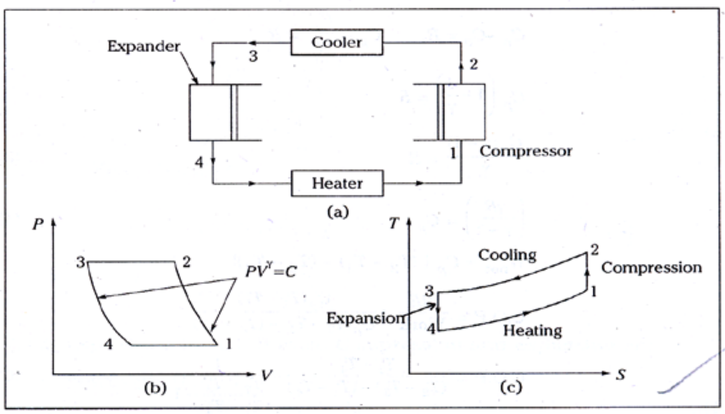
Process 2-3:

This high pressure, high temperature air, then enters a cooler where it is cooled at constant pressure to a low temperature.

Process 3-4: This high pressure, low temperature air is then expanded in an expander to lower pressure and temperature in a isentropic manner.At point 4, the temperature of the air will be lowest.

Process 4-1: This low temperature air is then passed through the heater coils where it absorbs heat from the space to be cooled namely the refrigerator and the air gets heated back to the initial temperature, but in the process, it cools the refrigerator. And the cycle repeats.

Air refrigeration system



Expression C.O.P when compression and expansion are Isentropic

Refrigeration Effect = Heat removed from the refrigerator

$$= C_p(T_1 - T_4) kJ/kg$$

$$Work\ \text{input} = W_C - W_E = \gamma\left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right) - \gamma\left(\frac{P_3V_3 - P_4V_4}{\gamma - 1}\right)$$

$$Work\ \text{input} = W_C - W_E = \left(\frac{\gamma}{\gamma - 1}\right)[R(T_2 - T_1) - R(T_3 - T_4)]$$

$$W_{net} = \left(\frac{\gamma R}{\gamma - 1}\right)[(T_2 - T_1) - (T_3 - T_4)]$$

$$But \qquad C_p = \frac{\gamma R}{\gamma - 1}$$

$$W_{net} = C_p[(T_2 - T_1) - (T_3 - T_4)]$$

Process 1 - 2 is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad ----(2)$$

Process 3 - 4 is isentropic

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad ----(3)$$

From (2) and (3)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$C.O.P = \frac{RE}{Work} = \frac{C_p(T_1 - T_4)}{C_p[(T_2 - T_1) - (T_3 - T_4)]}$$

$$C.O.P = \frac{(T_1 - T_4)}{[(T_2 - T_1) - (T_3 - T_4)]} = \frac{1}{\frac{T_2 - T_3}{T_1 - T_4} - 1} --(1)$$

$$\frac{T_2}{T_3} = \frac{T_1}{T_4}$$

$$\frac{T_2}{T_3} - 1 = \frac{T_1}{T_4} - 1$$

$$\frac{T_2 - T_3}{T_3} = \frac{T_1 - T_4}{T_4}$$

$$\frac{T_2 - T_3}{T_1 - T_4} = \frac{T_3}{T_4} \qquad -----(4)$$

From (1) and (4)

$$C.O.P = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$C.O.P = \frac{T_4}{T_3 - T_4}$$

For Polytropic process

Net work

$$W_{net} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)C_p\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

$$COP = \frac{T_4}{(T_3 - T_4)\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)}$$

Advantages of air refrigeration system

1. Air is cheap, easily available.
2. It is not flammable.
3. For a given capacity, weight of air refrigeration system is less compared to other system and hence it is widely used for aircraft cooling.

Disadvantages

1. Since heat removed by air consists only of sensible heat, weight of air required is high.
2. C.O.P of the system is low compared to other systems.

Problem 1

A cold storage is to be maintained at -5°C (268k) while the surroundings are at 35°C. the heat leakage from the surroundings into the cold storage is estimated to be 29kW. The actual C.O.P of the refrigeration plant is one third of an ideal plant working between the same temperatures. Find the power required to drive the plant.

Solution :-

$T_1 = 35°C = 308k \qquad T_2 = 5°C = 268k$

C.O.P of the ideal plant is nothing but

C.O.P based on carnot cycle.

$$\therefore \text{C.O.P ideal} = \frac{T_2}{T_1 - T_2}$$

$$= \frac{268}{308 - 268} = 6.7$$

$$Actual\ C.O.P = \frac{1}{3} ideal C.O.P$$
$$= \frac{1}{3} x6.7 = 2.233$$

Q2 = The heat removed from low temperature reservoir (cold storage) must be equal to heat leakage from surroundings to the cold storage(which is 29kw)

$$Q_2 = 29kW$$
$$Actual\ C.O.P = \frac{Q_2}{W}$$
$$\mathrm{W} = \frac{Q_2}{Actual\ C.O.P} = \frac{29}{2.233}$$
$$Power\ \text{required} = 12.98\ \text{kW}$$

Problem 2

A refrigeration machine of 6 tones capacity working on Bell coleman cycle has an upper limit pressure of 5.2 bar. The pressure and temperature at the start of the compression are I bar and 18°C respectively. The cooled compressed air enters the expander at 41°C. assuming both expansion and compression to be adiabatic with an index of 1.4.

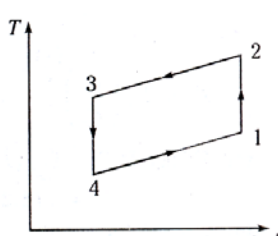
Calculate:-

(i) Co-efficient of performance.
(ii) Quantity of air circulated per minute.
(iii) Piston displacement of compressor and expander
(iv) Bore of compression and expansion cylinder when the unit runs at 240 rpm and is double acting with stroke length =200 mm
(v) Power required to drive the unit

Solution : -

$\mathrm{T}_1 = 18°C \quad \mathrm{P}_1 = 1\text{bar}$

$\mathrm{T}_3 = 41°C \ \mathrm{P}_2 = 5.2\text{bar}$



$$Work\ \text{input} = \mathrm{C_p}\left[(T_2 - T_1) - (T_3 - T_4)\right]$$
$$= 1.005\left[(466 - 291) - (314 - 196)\right] = 57 kJ/kg$$

$$C.O.P = \frac{\text{Re}\,griferation\ effect}{\text{Work input}}$$
$$= \frac{95.42}{57} = 1.67$$

$$\text{Re}\ frigeration\ \text{capacity} = 6\ \text{tons} = 6\text{x}3.5 = 21\text{kJ/s}$$

$$Mass\ \text{of air/sec} = \frac{\text{Re}\,griferation\ c\text{apacity}}{\text{R.E}}$$
$$= \frac{21}{95.42} = 0.22 kg/s$$
$$Power\ \text{required} = workdone\text{/kg of air x Mass of air/sec}$$
$$= 57\ \text{x}\ 0.22 = 12.54\text{kW}$$
$$\text{Mass of air/min} = 0.22\text{x}60 = 13.2\text{kg/min}$$

$$V_1 = \frac{mRT_1}{P_1} = \frac{13.2x0.287x291}{1x10^2} = 11 m^3/\min$$
$$Piston\ \text{displacement of compressor}\ \mathrm{V_1} = 11 m^3/\min$$
$$V_4 = \frac{mRT_4}{P_4} = \frac{13.2x0.287x196}{1x10^2} = 7.42 m^3/\min$$
$$Piston\ \text{displacement of expander}\ \mathrm{V_4} = 7.42 m^3/\min$$

$$But \quad V_1 = 2\frac{\pi}{4} d_1^2 LN$$
$$11 = 2\frac{\pi}{4} d_1^2 x0.2x240$$
$$d_1 = diameter\ \text{of compressor cylinder} = 0.38\text{m} = 38\text{cm}$$
$$V_4 = 2\frac{\pi}{4} d_2^2 LN$$
$$7.42 = 2\frac{\pi}{4} d_1^2 x0.2x240$$
$$d_1 = diameter\ \text{of expander cylinder} = 0.313\text{m} = 31.3\text{cm}$$

Problem3 An air refrigerator system operating on Bell Coleman cycle, takes in air from cold room at 268 K and compresses it from 1 bar to 5.5 bar the index of compression being 1.25. the compressed air is cooled to 300 K. the ambient temperature is 20ºC. Air expands in expander where the index of expansion is 1.35.

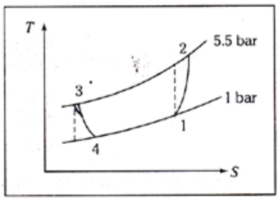
Calculate:

i) ***C.O.P of the system***
ii) ***Quantity of air circulated per minute for production of 1500 kg of ice per day at 0°C from water at 20ºC.***
iii) ***Capacity of the plant.***

Solution

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 268(5.5)^{\frac{1.25-1}{1.25}}$$

$$= 376.8\text{K}$$



$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 300\left(\frac{1}{5.5}\right)^{\frac{1.35-1}{1.35}} = 192.83\text{K}$$

$$\text{W}_\text{C} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)C_p(T_2 - T_1)$$

$$= \left(\frac{1.25}{1.25-1}\right)\left(\frac{1.4-1}{1.4}\right)1.005(376.8-268) = 156.2kJ/kg$$

$$\text{W}_\text{E} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)C_p(T_3 - T_4)$$

$$= \left(\frac{1.35}{1.35-1}\right)\left(\frac{1.4-1}{1.4}\right)1.005(300-192.83) = 118.69kJ/kg$$

$$Network = W_C - W_E = 156.2 - 118.69 = 37.5kJ/kg$$

$$R.E = C_p(T_1 - T_4) = 1.005(268-192.83) = 75.54kJ/s$$

$$C.O.P = \frac{RE}{work} = \frac{75.54}{37.5} = 2$$

$$Heat\ \text{extracted/kg of ice} = \mathrm{C_{pw}}(20-0)+L$$

$$= 4.187(20)+335 = 418.74\text{kJ/kg}$$

$$\text{Mass of ice produced/sec} = \frac{1500}{24\text{x}3600} = 0.0173 kg/s$$

$$Actual\ heat\ \text{extracted/sec} = 418.74\text{x}0.0173$$

$$\text{or Refrigeration capacity} = 7.26\text{kJ/s} = \frac{7.26}{3.5} = 2.02 tons$$

$$Mass\ \text{flow rate} = \frac{\text{Refrigeration Capacity}}{\text{Refrigeration efect}} = \frac{7.26}{75.54}$$

$$= 0.096 kg/s$$

Problem 4

An air refrigeration system is to be designed according to the following specifications

Pressure of air at compressor inlet=101kPa

Pressure of work at compressor outlet=404kPa

Pressure loss in the inter cooler=12kPa

Pressure loss in the cold chamber=3kPa

Temperature of air at compressor inlet=7°

Temperature of air at turbine inlet=27°

Isentropic efficiency of compressor =85%

Isentropic efficiency of turbine =85%

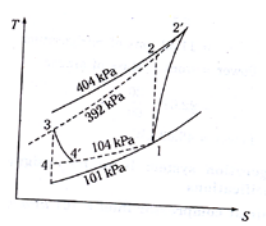
Determine

i) C.O.P of cycle
ii) Power required to produce 1 ton of refrigeration
iii) Mass flow rate of air required for 1 ton of refrigeration

Solution :-

$\mathrm{T_1} = -7°C \quad \mathrm{P_1} = 101\text{kPa}$

$\mathrm{T_3} = 27°C\ \eta_\mathrm{T} = 0.85; \eta_\mathrm{C} = 0.85$



$$\text{Process 1-2 is isentropic, Hence } T'_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 266\left(\frac{404}{101}\right)^{\frac{1.4-1}{1.4}} = 395.4\text{K}$$

$$\eta_C = \frac{T_2 - T_1}{T'_2 - T_1} \ \text{or} T'_2 - T_1 = \frac{395.4 - 266}{0.88}$$

$$T'_2 = 418.2k$$

$$P_4 - P_1 = 0.03P_1 \quad \therefore P_4 = 1.03P_1 = 1.03x101 = 104kPa$$

$$P_2 - P_3 = 0.03P_2 \quad \therefore P_3 = 0.97P_2 = 0.97x404 = 392kPa$$

$$\text{Pr}\,ocess\ 3\text{-}4\ \text{is isentropic,} \quad \therefore T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300\left(\frac{104}{392}\right)^{\frac{1.4-1}{1.4}} = 202.3\text{K}$$

$$\eta_E = \frac{T_3 - T'_4}{T_3 - T_4} \ \therefore T'_4 = T_3 - \eta_T\left(T_3 - T_4\right)$$

$$T'_4 = 300 - 0.85x[300 - 205.3] = 216.53k$$

$$\text{Re}\ frigeration\ \text{effect/kg of air} = \text{C}_\text{p}\left(T_1 - T_4\right)$$

$$= 1.005\text{x}[266 - 216.53] = 50.47\text{kJ/kg}$$

$$C\text{ompressor work/kg of air} = \text{C}_\text{p}\left(T'_2 - T_1\right)$$

$$= 1.005\text{x}[418.2 - 266] = 152.96\text{kJ/kg}$$

$$T\text{urbine work/kg of air} \quad \text{W}_\text{T} = \text{C}_\text{p}\left(T_3 - T_4'\right)$$

$$= 1.005\text{x}[300 - 216.53] = 84.9\text{kJ/kg}$$

$$\text{Net work Input/kg of air W}_\text{net} = \text{W}_\text{C} - \text{W}_\text{T}$$

$$= 152.96 - 80.9 = 72.06kJ/kg$$

$$C.O.P = \frac{RE}{Work} = \frac{46.73}{72.06} = 0.73$$

$$Power\ \text{required per tons of refergeration}$$

$$= \frac{\text{Refrigeration capacity}}{\text{C.O.P}}$$

$$\text{Refrigeration capacity} = 1\ \text{ton} = 3.5\text{kJ/s}$$

$$\text{Mass of air} = \frac{\text{Refrigeration capacity}}{\text{RE}}$$

$$= \frac{3.5}{50.47} = 0.075kg/s$$

$$Power = W_{net} xmassofair/\sec = 72.06x0.075 = 5.42kW$$

Problem 1

Moist air at 30°C,1.01325 bar has a relative humidity of 80%. Determine without using the psychrometry chart

1) Partial pressures of water vapour and air
2) Specific humidity
3) Specific Volume and
4) Dew point temperature (V.T.U. July2004)

Solution: At 30°C from table $p_{\upsilon s} = 4.2461 kPa$

$$\phi = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$p_\upsilon = 0.8x4.2461 = 3.397 kPa$$

$$\omega = \frac{0.622 p_\upsilon}{p - p_\upsilon} = 0.622x\frac{3.397}{101.325 - 3.397}$$

$$= 0.213 \text{ kg/kg of dry air.}$$

Corresponding to Pv =3.397 kPa from tables, we get dew point temperature = 28.9°C

Problem 2:

Atmospheric air at 101.325 kPa ha 30°C DBT and 15°C DPT. Without using the pschrometric chart, using the property values from the table, Calculate

1. Partial pressure of air and water vapour
2. Specific humidity
3. Relative humidity
4. Vapour density and
5. Enthalpy of moist air

Solution:

$p = 101.325 kpa = 1.01325 bar$

$DBT = 30°C,$

$\text{DPT} = 15°C$

From table

Corresponding to DBT = 30°C, we have $p_{\upsilon s} = 0.042461 bar$

Corresponding to DPT = 15°C, we have $p_\upsilon = 0.017051 bar$

Partial pressure of air $= p - p_\upsilon = 1.01325 - 0.017051$

$$= 0.984274 \text{ bar}$$

$$Specific\ humidity = 0.622\frac{p_\upsilon}{p_a} = \frac{0.622x0.017051}{0.984274}$$

$$= 0.01077\text{kJ/kg of dry air}$$

$$\text{Re}\,lative\ humidity = \frac{p_\upsilon}{p_{\upsilon s}} = \frac{0.017051}{0.042461} = 0.4015$$

$$= 40.15\%$$

$$\text{Enthalphy} = 1.005\text{t}_{\text{db}} + \omega(2500 + 1088t_{db})$$

$$= 1.005\text{x}30 + 0.010775(2500 + 1.88\text{x}30)$$

$$= 57.69\text{kj/kg of dry air}$$

$$Specific\ \text{volume of dry air}, \upsilon_a = \frac{RT}{P}$$

$$= \frac{0.2872\text{x}303}{0.98425x100} = 0.874m^3/kg$$

$$Vapour\ \text{density}\ \rho_w = \frac{\omega}{\upsilon_a} = \frac{0.010775}{0.847} = 0.12kg/m^3$$

Problem 3:

Air at 30°C DBT and 25°C WBT is heated to 40°C. if the air is 300 m3/min, find the amount of heat added/min and RH and WBT of air. Take air pressure to be 1 bar

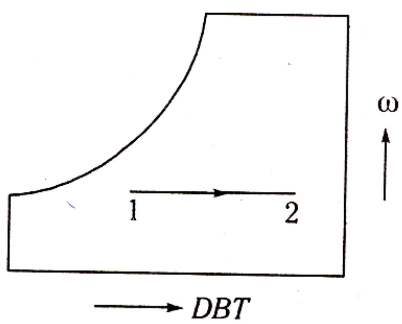
Solution:

At 25°C WBT from tables pwswbt=0.03166 bar

$$\therefore\ \text{p}_\upsilon = (P_{VS})_{wbt} - \frac{(p - p_{\upsilon swbt})(t_{db} - t_{wb})}{1547 - 1.44t_{wb}}$$

$$= 0.03166 - \frac{(1 - 0.03166)(30 - 25)}{1547 - 1.44x25}$$

$$= 0.0284\ \text{bar}$$



$$\omega_1 = 0.622\frac{p_\upsilon}{p - p_\upsilon}$$

$$= 0.622\left(\frac{0.0284}{1 - 0.0284}\right)$$

$$= 0.0179kJ/kg\ \text{of dry air}$$

$At\ 40°C\ DBT$

$P_{VS} = 0.07375 bar$

$During\ sensible\ heating\ \omega$ and p_{υ} $remain$ constant

$p_{\upsilon} = 0.0284 bar$

$$RH = \phi = \frac{p_{\upsilon}}{p_{\upsilon s}} = \frac{0.0284}{0.07375}$$

$$= 0.385 = 38.5\%$$

$$H_2 = 1.005x40 + 0.0179(2500 + 1.88x40)$$

$$= 86.29 \text{kJ/kg of dry air}$$

$$Weight \text{ of } 300\text{m}^3 / \min ofair = \frac{(p - p_{\upsilon})V}{RT}$$

$$= \frac{(1\text{-}0.0284)\text{x}300\text{x}10^2}{0.287x303} = 335.18 kg / \min$$

$\therefore Heat$ added/min $= 335.18(86.29 - 76) = 3449$kJ/min

From chart WBT $= 27.2°$C

Problem 4:

One stream of air at 5.5m3/min at 15°C and 60% RH flows into another stream of air at 35m3/min at 25°C and 70%RH, calculate for the mixture

1) Dry bulb temperature, 2) Wet bulb temperature 3) Specific Humidity and 4) Enthalpy

Solution:

For air at 15°C and 60%RH, V=5.5m3/min

$$\therefore\ p_{\upsilon s} = 0.017051 bar$$

$$RH = \phi = \frac{p_{\upsilon}}{p_{\upsilon s}}$$

$$\therefore\ p_{\upsilon} = 0.6x0.017051 = 0.01023 bar$$

$$Mass \text{ of air} = \frac{(\text{p} - \text{p}_{\upsilon})V}{RT} = \frac{(1.01325 - 0.01023)x10^2 x5.5}{0.287x288}$$

$$m_1 = 6.672 kg / \min$$

$$\omega_1 = \frac{0.622 p_{\upsilon}}{(\text{p} - \text{p}_{\upsilon})} = \frac{0.622x0.01023}{(1.01325 - 0.01023)}$$

$$= 0.006343 kg / kg \text{ of dry air}$$

$$H_1 = 1.005t_{db} + \omega_1(2500 + 1.88t_{db})$$
$$= 1.008x18 + 0.006343(2500 + 1.88x15)$$
$$= 34.12 \text{J/kg of dry air}$$

For air at 25°C and 70% RH, $\quad V = 35m^3/\min$

$$P_{\upsilon s} = 0.03169 bar$$

$$\phi = RH = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$p_\upsilon = 0.03169x0.7 = 0.02218 bar$$

$$H_1 = 1.005t_{db} + \omega_1(2500 + 1.88t_{db})$$
$$= 1.008x18 + 0.006343(2500 + 1.88x15)$$
$$= 34.12 \text{J/kg of dry air}$$

For air at 25°C and 70% RH, $\quad V = 35m^3/\min$

$$P_{\upsilon s} = 0.03169 bar$$

$$\phi = RH = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$p_\upsilon = 0.03169x0.7 = 0.02218 bar$$

$$Mass \text{ of air} = \frac{(1.01325 - 0.02218 x10^2 x35)}{0.287x298}$$

$$m_2 = 40.55 kg.\min$$

$$\omega_2 = \frac{0.622x0.02218}{(1.01325 - 0.02218)} = 0.01392 kg/kg \text{ of dry air}$$

$$H_2 = (1.005x25) + 0.01392(2500 + 1.88x25)$$

$$H_2 = 60.59 kJ/kg \text{ of dry air}$$

Mass of dry air/Unit mass of moist air

$$m_{a1} = \frac{m_1}{1 + \omega_1} = \frac{6.672}{1 + 0.006343} = 6.6299$$

$$\text{Since } m_{a2} = \frac{m_2}{1 + \omega_2} = \frac{40.55}{1 + 0.01392} = 39.993$$

Then enthalpy of the mixed air,

$$H_{mix} = \frac{m_{a1}(H_1) + m_{a2}(H_2)}{m_1 + m_2}$$
$$= \frac{6.6299(34.12) + 39.993(60.56)}{6.672 + 40.55)}$$
$$= 55.96 \text{ kJ/kg of dry air}$$

Specific Humidity of the mixed air,

$$\omega_{\text{mix}} = \frac{m_{a1}(\omega_1) + m_{a2}(\omega_2)}{m_1 + m_2}$$

$$= \frac{(6.6299\text{x}0.006343) + (39.993\text{x}0.01932)}{6.672 + 40.55}$$

$$= 0.01268\text{kg/kg of dry air}$$

$$\text{But H}_{\text{mix}} = 1.005t_{db} + \omega_{mix}(2500 + 1.88t_{db})$$

$$55.96 = 1.005xt_{db} + 0.01234(2500 + 1.88t_{db})$$

$$t_{db} = 24.42°C$$

$$DBT \text{ of the mixture} = 24.42°\text{C}$$

$$\text{From chart WBT} = 19°\text{C}$$

$$\text{RH} = 67\%$$

Problem 5:

An air conditioning system is designed under the following conditions

Outdoor conditions: 30°CDBT, 75% RH

Required indoor conditions: 22°CDBT,70% RH

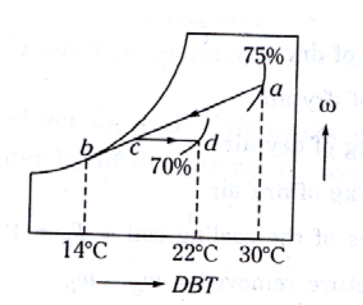
Amount of Free air circulated 3.33 m3/s

Coil dew point temperature DPT=14°

The required condition is achieved first by cooling and dehumidification and then by heating. Estimate

1) The capacity of the cooling coil in tons of refrigeration
2) *Capacity of the heating coil in kW*
3) *The amount of water vapour removed in kg/hr*

Solution:



Locate point 'a' 30°C DBT, 75%RH out door condition

Locate point 'd' 22°C DBT, 70%RH required condition

Locate point 'b' 14°C DPT, coil surface temperature

Join ab

at d, draw a horizontal line to cut the

line ab at point c.

ac $\rightarrow$ cooling and dehumidification

cd $\rightarrow$ heating

From chart

$H_a = 83 kJ/kg$ of air

$H_b = 40 kJ/kg$ of air

$H_d = 53 kJ/kg$ of air

$H_c = 48 kJ/kg$ of air

$W_a = 0.0202 kg/kg$ of dry air

$W_c = W_d = 0.0118 kg/kg$ of dry air

$V_{sa} = 0.88 m^3/kg$

$$\text{Mass of air} = \frac{V}{V_a} = \frac{3.33}{0.88} = 3.78 kg/s$$

$$\text{Capacity of cooling coil} = \frac{m_a(H_a - H_c)}{3.5}$$

$$= \frac{3.78(83-48)}{3.5} = 37.84 \text{ tons of refrigeration}$$

$$\text{Capacity of heating coil} = m_a(H_d - H_c)$$

$$= 3.78(53 - 48) = 18.92 \text{kW}$$

$$\text{Amount of water vapour removed} = m_a(\omega_a - \omega_d)3600$$

$$= 3.78(0.0202 - 0.0118)3600$$

$$= 114.3 \text{kg/hr}$$